Numerical Simulation of 3D Cavitation behind a Disk Cavitator Using OpenFOAM

Presenter: Amin Rahimi

Dr. Ehsan Roohi
Dr. Mohamad Javad Maghrebi
Outline

- Cavitation Phenomenon
- Turbulence Modeling
  - Cavitation modeling (VOF)
- Mass Transfer
- Simulation setups
- Results
- Conclusion
Cavitation Phenomenon

- Cavitation on Propeller
- Cavitation on Torpedo
- Cavitation behind Disk
Main models in simulating the cavitating flow

- Turbulence modeling
- Multiphase Modeling (VOF)
- Mass Transfer Modeling
LES turbulence model:

*The large energy-containing structures resolve and the smaller, more isotropic sub-grid structures are modeled*

*Applying low-pass filtering, using a pre-defined filter kernel function $G = G(x, \Delta)$*

*LES in contrast with RANS approaches*
Filtered Navier-Stokes and continuity equation are usually provided by convolving all dependent variables with \( G = G(x, \Delta) \)

\[
G_B(r) = \begin{cases} 
  \frac{1}{\Delta}, & \text{if } r \leq \Delta/2 \\
  0, & \text{if } r > \Delta 
\end{cases}
\]

\[
\int_{-\infty}^{\infty} G_B(r) \, dr = 1
\]

\[
\begin{align*}
\partial_t (\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \times \vec{v}) &= -\nabla p + \nabla \cdot (\vec{s} - \vec{B}) \\
\partial_t \rho + \nabla \cdot (\rho \vec{v}) &= 0
\end{align*}
\]

- \( \vec{B} \) is the SGS stress tensor
- Only \( \vec{B} \) needs to be modeled

Eliminate eddies smaller than the filter width (\( \Delta \))
Turbulence modeling

eddy or sub grid viscosity models

\[ B = \frac{2}{3} \bar{\rho} k I - 2 \mu_k \tilde{D}_D \]

- One-Equation Eddy-Viscosity Model (OEEVM):

\[ \partial_t (\bar{\rho} k) + \nabla \cdot (\bar{\rho} k \bar{v}) = -B \cdot \tilde{D} + \nabla \cdot (\mu \nabla k) + \bar{\rho} \varepsilon \]

K (SGS kinetic energy)

\[ \varepsilon = \frac{c_\varepsilon k^{3/2}}{\Delta} \quad \text{AND} \quad \mu_k = c_k \bar{\rho} \Delta \sqrt{k} \]
Turbulence modeling

$k - \omega$ SST turbulence model:

Turbulence Kinetic Energy:

\[
\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_j} (\rho ku_j) = \frac{\partial}{\partial x_j} \left( \left( \mu + \frac{\mu_t}{\sigma_{k3}} \right) \frac{\partial k}{\partial x_j} \right) \\
+ \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta^* \rho k \omega.
\]

Specific dissipation rate:

\[
\rho \frac{\partial \omega}{\partial t} + \rho u_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \mu + \frac{\mu_t}{\sigma_{\omega3}} \right) \frac{\partial \omega}{\partial x_j} \right) \\
+ \frac{\omega}{k} \left( \alpha_3 \tau_{ij} \frac{\partial u_i}{\partial x_j} - \beta_3 \rho k \omega \right) \\
+ (1 - F_1) \frac{2 \rho}{\omega \sigma_{\omega2}} \frac{1}{\omega \sigma_{\omega2}} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j},
\]
Turbulence modeling

Where the coefficients of the model are a linear combination of the corresponding coefficients of the $k-\omega$ and $k-\varepsilon$ modified models as:

$$\psi = F_1 \psi_{k\omega} + (1 - F_1) \psi_{k\varepsilon}.$$

$k-\omega$: $\alpha_1 = 5/9$, $\beta_1 = 3/40$, $\sigma_{k_1} = 2$,

$\sigma_{\omega_1} = 2$, $\beta^* = 9/100$,

$k-\varepsilon$: $\alpha_2 = 0.44$, $\beta_2 = 0.0828$, $\sigma_{k_2} = 1$,

$\sigma_{\omega_2} = 1/0.856$, $C_\mu = 0.09$. 
Cavitation modeling (VOF)

volume fraction, $\alpha$

$$\alpha = \frac{V_v}{V_v + V_1}$$

one fluid, two-phase mixture

$$\rho = \alpha \rho_v + (1 - \alpha) \rho_l$$

$$\mu = \alpha \mu_v + (1 - \alpha) \mu_l$$

Transport equation:

$$\partial_t \alpha + \nabla \cdot (\alpha \vec{v}) = \dot{m} / \rho_v$$
Mass Transfer Modeling

- Kunz model
- vaporization occurs when the pressure is below the vapor pressure

\[
\begin{align*}
\dot{m}^+ &= A^+ \rho_v (1 - \alpha) \frac{\min [0, \bar{p} - p_v]}{\frac{1}{2} \rho_l U_\infty^2} \\
\dot{m}^- &= A^- \rho_v (1 - \alpha) \alpha^2 \\
\dot{m} &= \dot{m}^+ - \dot{m}^-
\end{align*}
\]
OpenFoam structure

Big library → Applications

- MultiPhaseSolvers

interPhaseChangeFoam
Simulation setups for 3D simulation

- Disk: $\sigma = 0.245, \text{Re} = 5.4 \times 10^5, U_\infty = 20 \text{ (m/s)}$
Structured Mesh (gambit)

There is 79000 cell in the total domain
Results

- 3D cavitation behind a disk

\[ \sigma = 0.2 \]

\[ \sigma = 0.245 \]
Results

- Side view and 3D view of vapor phase (cavity region) for $\sigma = 0.2$ with LES turbulence model
Results

- Side view and 3D view of vapor phase (cavity region) for $\sigma = 0.2$ with $\kappa - \omega$ SST turbulence model
Results

Section of cavitating flow for $\sigma = 0.245$

Comparing the contour of vapor phase (cavity region) using two type of turbulence models, $t=50$ ms, $\sigma = 0.2$
# Results

## Computed parameters

**Table 1: Computed parameters at \( \sigma = 0.2 \)**

<table>
<thead>
<tr>
<th>Method</th>
<th>Simulation LES</th>
<th>Simulation ( \kappa - \omega ) SST</th>
<th>Richardt’s Theory [6]</th>
<th>Exp. [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L/d )</td>
<td>8.57</td>
<td>8.16</td>
<td>5.78</td>
<td>7.46</td>
</tr>
<tr>
<td>( D/d )</td>
<td>2.4</td>
<td>2.27</td>
<td>2.3</td>
<td>2.66</td>
</tr>
<tr>
<td>( C_D )</td>
<td>1.03</td>
<td>1.03</td>
<td>1.00</td>
<td>1.01</td>
</tr>
</tbody>
</table>

**Table 2: Computed parameters at \( \sigma = 0.245 \)**

<table>
<thead>
<tr>
<th>Method</th>
<th>Simulation LES</th>
<th>Simulation ( \kappa - \omega ) SST</th>
<th>Richardt’s Theory [6]</th>
<th>Exp. [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L/d )</td>
<td>6.08</td>
<td>5.18</td>
<td>4.42</td>
<td>5.99</td>
</tr>
<tr>
<td>( D/d )</td>
<td>2.2</td>
<td>2.08</td>
<td>2.13</td>
<td>2.36</td>
</tr>
<tr>
<td>( C_D )</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Results

Richardt’s Relations

\[
\frac{L}{d} = \frac{\sigma + 0.008}{\sigma(1.7\sigma + 0.066)} \left( \frac{d}{D} \right)
\]

\[
\frac{d}{D} = \left[ \frac{C_D}{\sigma\left(1 - 0.132\sigma^{0.5}\right)} \right]^{0.5}
\]

\[
C_D = C_{D_0} \left(1 + \sigma\right)
\]
Conclusion

- Cavitating flow simulation using LES $\kappa-\omega$ SST/VOF and Kunz mass transfer model

- Agreements between experimental and numerical results
  
  - Cavitation shape
  - Drag coefficient
  - LES in contrast with $\kappa-\omega$ SST
Thank you