12.3 Application to Supersonic Airfoils

Equation (12.15) is very handy for estimating the lift and wave drag for thin supersonic airfoils, such as sketched in Figure 12.3. When applying Equation (12.15) to any surface, one can follow a formal sign convention for θ, which is different for regions of left-running waves (such as above the airfoil in Figure 12.3) than for regions of right-running waves (such as below the airfoil in Figure 12.3). This sign convention is

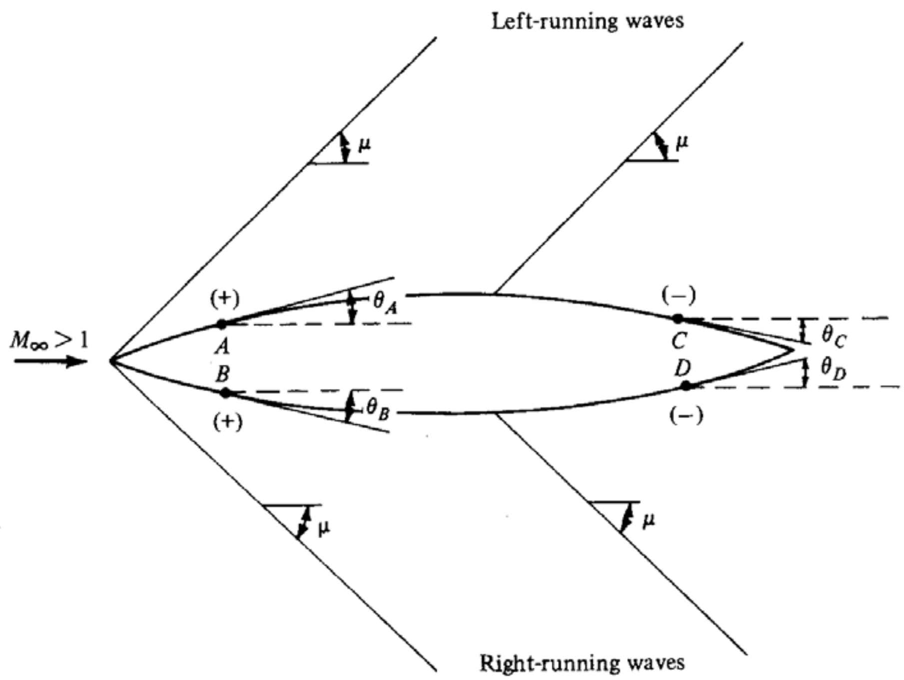

Figure 12.3 Linearized supersonic flow over an airfoil.

developed in detail in Reference 21. However, for our purpose here, there is no need to be concerned about the sign associated with θ in Equation (12.15). Rather, keep in mind that when the surface is inclined *into* the freestream direction, linearized theory predicts a positive C_p. For example, points A and B in Figure 12.3 are on surfaces inclined into the freestream, and hence $C_{p,A}$ and $C_{p,B}$ are positive values given by

$$C_{p,A} = \frac{2\theta_A}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p,B} = \frac{2\theta_B}{\sqrt{M_\infty^2 - 1}}$$

In contrast, when the surface is inclined *away from* the freestream direction, linearized theory predicts a negative C_p. For example, points C and D in Figure 12.3 are on surfaces inclined away from the freestream, and hence $C_{p,C}$ and $C_{p,D}$ are negative values, given by

$$C_{p,C} = -\frac{2\theta_C}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p,D} = -\frac{2\theta_D}{\sqrt{M_\infty^2 - 1}}$$

In the above expressions, θ is always treated as a positive quantity, and the sign of C_p is determined simply by looking at the body and noting whether the surface is inclined into or away from the freestream.

With the distribution of C_p over the airfoil surface given by Equation (12.15), the lift and drag coefficients, c_l and c_d, respectively, can be obtained from the integrals given by Equations (1.15) to (1.19).

Let us consider the simplest possible airfoil, namely, a flat plate at a small angle of attack α as shown in Figure 12.4. Looking at this picture, the lower surface of the

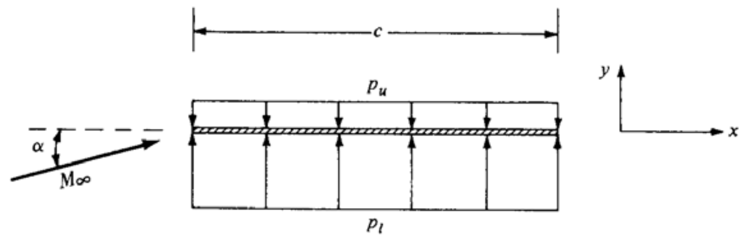

Figure 12.4 A flat plate at angle of attack in a supersonic flow.

plate is a compression surface inclined at the angle α into the freestream, and from Equation (12.15),

$$C_{p,l} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \quad [12.16]$$

Since the surface inclination angle is constant along the entire lower surface, $C_{p,l}$ is a constant value over the lower surface. Similarly, the top surface is an expansion surface inclined at the angle α away from the freestream, and from Equation (12.15),

$$C_{p,u} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \quad [12.17]$$

$C_{p,u}$ is constant over the upper surface. The normal force coefficient for the flat plate can be obtained from Equation (1.15):

$$c_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})\, dx \quad [12.18]$$

Substituting Equations (12.16) and (12.17) into (12.18), we obtain

$$c_n = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \frac{1}{c}\int_0^c dx = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \quad [12.19]$$

The axial force coefficient is given by Equation (1.16):

$$c_a = \frac{1}{c}\int_{\text{LE}}^{\text{TE}} (C_{p,u} - C_{p,l})\, dy \quad [12.20]$$

However, the flat plate has (theoretically) zero thickness. Hence, in Equation (12.20), $dy = 0$, and as a result, $c_a = 0$. This is also clearly seen in Figure 12.4; the pressures act normal to the surface, and hence there is no component of the pressure force in the x direction. From Equations (1.18) and (1.19),

$$c_l = c_n \cos\alpha - c_a \sin\alpha \quad [1.18]$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha \quad [1.19]$$

and, along with the assumption that α is small and hence $\cos\alpha \approx 1$ and $\sin\alpha \approx \alpha$, we have

$$c_l = c_n - c_a\alpha \quad \textbf{[12.21]}$$

$$c_d = c_n\alpha + c_a \quad \textbf{[12.22]}$$

با توجه به توضیحات فوق، سوال زیر قابل حل است:

اگر معادله خط کمبر یک ایرفویل نازک به شکل $\frac{y}{c} = -\varepsilon(\frac{x}{c})^{۳}$ و $\varepsilon <<۱$ باشد به کمک تئوری جریان خطی در جریان مافوق صوت مقدار c_a (ضریب نیروی محوری) و c_n (ضریب نیروی عمودی) به ترتیب کدام است؟

۱) $c_a = \frac{۴\varepsilon^{۲}}{\sqrt{m_\infty^{۲}-۱}}$, $c_n = \frac{۴\varepsilon}{\sqrt{m_\infty^{۲}-۱}}$

۲) $c_a = \frac{۹\varepsilon^{۲}}{۲\sqrt{m_\infty^{۲}-۱}}$, $c_n = \frac{۵\varepsilon}{۳\sqrt{m_\infty^{۲}-۱}}$

۳) $c_a = \frac{۱۶\varepsilon^{۲}}{۳\sqrt{m_\infty^{۲}-۱}}$, $c_n = \frac{۱۲\varepsilon}{\sqrt{m_\infty^{۲}-۱}}$

۴) $c_a = \frac{۳۶\varepsilon^{۲}}{۵\sqrt{m_\infty^{۲}-۱}}$, $c_n = \frac{۴\varepsilon}{\sqrt{m_\infty^{۲}-۱}}$

برای سطح پایینی ایرفویل داریم:

$$C_{p,A} = \frac{2\theta_A}{\sqrt{M_\infty^2 - 1}}$$

برای سطح بالایی ایرفویل:

$$C_{p,C} = -\frac{2\theta_C}{\sqrt{M_\infty^2 - 1}}$$

که تتا همان شیب ایرفویل است(که برابر با مشتق کمبر است):

$$dy = -3\varepsilon(\frac{x}{c})^2 dx$$

$$\theta = tan\theta = \frac{dy}{dx} = -3\varepsilon(\frac{x}{c})^2$$

با محاسبه تتا ضرایب فشار محاسبه میشوند و با استفاده از روابط زیر:

$$c_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})\,dx$$

$$c_a = \frac{1}{c}\int_{\mathrm{LE}}^{\mathrm{TE}} (C_{p,u} - C_{p,l})\,dy$$

از انجا که می دانیم رابطه ی ضریب فشار به صورت زیر است داریم:

$$C_P = \frac{2\theta}{\lambda}$$

$$\lambda = \sqrt{M_\infty^2 - 1}$$

با جایگذاری تتا در بالا داریم:

$$C_P = \frac{-6\varepsilon}{C^2} * \frac{X^2}{\lambda}$$

که برای بالا و پایین ایرفویل اینگونه است:

$$C_{P,U} = \frac{2\theta}{\lambda} \qquad C_{P,L} = -\frac{2\theta}{\lambda}$$

حال با محاسبه انتگرال ها داریم:

$$C_n = \frac{1}{c}\int_0^c -\frac{4}{\lambda}\frac{dy}{dx}dx = \frac{1}{c}\int_0^c \frac{12\varepsilon x^2}{\lambda c^2}dx = \frac{4\varepsilon}{\lambda}$$

$$C_a = \frac{1}{c}\int_0^c \frac{2}{\lambda}\left[\frac{dy}{dx}^2 + \frac{dy}{dx}^2\right]dx = \frac{2}{\lambda c}\int_0^c \frac{18\varepsilon^2 x^4}{c^4}dx = \frac{36\varepsilon^2}{5\lambda}$$

لذا گزینه صحیح گزینه ۴ می‌باشد.