بسمه تعالی

تمرین (بخش تئوری جریان مافوق صوت خطی شده) مهلت تحویل: ۱۳۹۴/۱/۲۵

Problem 1. Supersonic Thin Airfoil Theory. The main objective of this problem is to understand the effects of angle of attack, thickness, and camber distributions of a supersonic thin airfoil on its performance (lift, drag, and moment coefficients, as well as the location of the center of pressure and the aerodynamic center).

Consider a family of supersonic airfoils (for illustration purposes only: you would normally design them differently) given by the following camber and thickness distributions. The camberline is that of the NACA four-digit series of airfoils that you used in PS1, and is given by the following two parabolas:

$$\bar{Y}(x) = \begin{cases} \frac{\epsilon x}{p^2}\left(2p - \frac{x}{c}\right) & \text{for } 0 < \frac{x}{c} < p \\ \frac{\epsilon(c-x)}{(1-p)^2}\left(1 + \frac{x}{c} - 2p\right) & \text{for } p < \frac{x}{c} < 1, \end{cases}$$

where ϵ is the maximum camber ratio and p is the chordwise location of the maximum camber. The thickness distribution is parabolic and given by:

$$T(x) = 4\tau\frac{x}{c}(c-x),$$

where τ is the thickness-to-chord ratio. As usual, you can construct the upper and lower surfaces of the airfoil by simply adding/subtracting one half the thickness to/from the camberline (vertically: no need to add the thickness normal to the camberline):

$$\begin{aligned} y_u(x) &= \bar{Y} + \frac{1}{2}T(x) \\ y_l(x) &= \bar{Y} - \frac{1}{2}T(x). \end{aligned}$$

Using thin airfoil theory, derive closed-form expressions (using Mathematica or MATLAB's Symbolic Toolbox is probably a good idea, although it can also be done by hand) for:

- The coefficient of lift, C_l, of the airfoil, as a function of ϵ, p, τ, and α.
- The coefficient of wave drag, C_d, of the airfoil, as a function of ϵ, p, τ, and α.
- The coefficient of moment about the leading edge, $C_{m_{le}}$, of the airfoil, as a function of ϵ, p, τ, and α.

Use these expressions to compute the location of the center of pressure, x_{cp}, for the family of airfoils. Where is the aerodynamic center located? Is it a function of any of the parameters in this problem (ϵ, p, τ, and α)?

Finally, for an airfoil of this family with 1% maximum camber located at the quarter chord, and 2% thickness, please plot the variation of C_l, C_d, x_{cp}, and $C_{m_{le}}$ as a function of angle of attack

in what you would consider a logical range of α. What is this airfoil's optimum lift-to-drag ratio and at what α and C_l is it achieved?