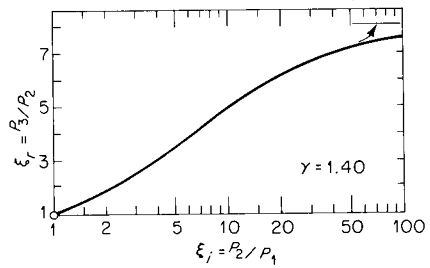

Figure 8.36
Pressure ratio of shock reflected as a function of the pressure ratio of the incident shock (perfect gas, $\gamma = 1.40$).

This relation is plotted in Fig. 8.36. The reader can verify that ξ_r has the asymptotic value $\xi_r = (3\gamma - 1)/(\gamma - 1)$ as $\xi_i \to \infty$ and that $\xi_r < \xi_i$ for all $\xi_i > 1$.

Additional relations of this kind are given by *Polachek and Seeger* [1958].

EXAMPLE 8.6 SHOCK INTERSECTION

Air at $P_1 = 1$ atm and $T_1 = 300$ K travels with velocity $u_1 = 500$ ft/s in a rigid pipe of constant area. The flow is brought to rest by a leftward-traveling shock A. At the same time, a rightward-traveling shock B with pressure ratio 5 overtakes the flow. Find the conditions in the neighborhood of the shock collision point (see Fig. 8.37).

The sound speed c_1 is found to be $c_1 = \sqrt{\gamma RT_1} = 1{,}139$ ft/s. Then with the given information and the shock tables, the values listed in Table 8.3 are found for fields 2 and 3. To find conditions in fields 4 and 5 we match velocities and pressure. One way to proceed is to assume a final pressure and calculate u_4 and u_5; if they do not match, the process is repeated until they do. [An alternative is to apply Eq. (8.64) directly, with $P_4 = P_5$ and $u_4 = u_5$.] A graphical plot may be helpful. For example, if we guess $P_4 = P_5 = 10$ atm, we find from the shock tables

$$u_5 = +1{,}254 \text{ ft/s}$$
$$u_4 = +1{,}822 \text{ ft/s}$$

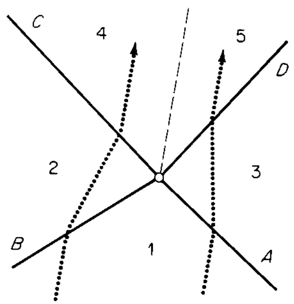

Figure 8.37

Table 8.3

Field	u, ft/s	P, atm	c, ft/s	M_s
1	500	1.00	1,139	$M_A = 1.297$
2	2,047	5.00	1,517	$M_B = 2.105$
3	0	1.80	1,240	$M_C = 1.227$
4	1,527	7.95		$M_D = 1.982$
5	1,527	7.95		

This guess was too high. The final result is

$$P_4 = P_5 = 7.95 \text{ atm}$$
$$u_4 = u_5 = +1{,}527 \text{ ft/s}$$

An entropy discontinuity has been developed. To demonstrate this, we can, for example, calculate the final densities and find

$$\rho_5 = 3.92\rho_1$$
$$\rho_4 = 3.99\rho_1$$

EXAMPLE 8.7 SHOCK INCIDENT ON AN OPEN END

A shock travels down a tube containing perfect gas ($\gamma = 1.40$) and approaches an open end. The fluid ahead of the shock is at rest. Find the flow conditions within the tube after the shock reaches the exit plane, for three different shock Mach numbers M_{1n},

Case a: $M_{1n} = 1.25$
Case b: $M_{1n} = 1.50$
Case c: $M_{2n} = 2.50$

These three cases have been calculated and are illustrated in Fig. 8.38.

Case a. The shock tables give directly that $u_2 = 0.375c_1$, $c_2 = 1.077c_1$, and $P_2 = 1.656P_1$. Assuming that the outflow is subsonic, $P_3 = P_1$; then

$$\frac{c_3}{c_2} = \left(\frac{P_3}{P_2}\right)^{(\gamma-1)/2\gamma} = \left(\frac{1}{1.656}\right)^{1/7} = 0.9305$$

This gives $c_3 = 1.002c_1$. From the constancy of the Riemann invariant between fields 2 and 3, find

$$\frac{u_3}{c_1} = \frac{u_2}{c_1} + 5\left(\frac{c_2}{c_1} - \frac{c_3}{c_1}\right) = 0.375 + 0.375 = 0.750$$

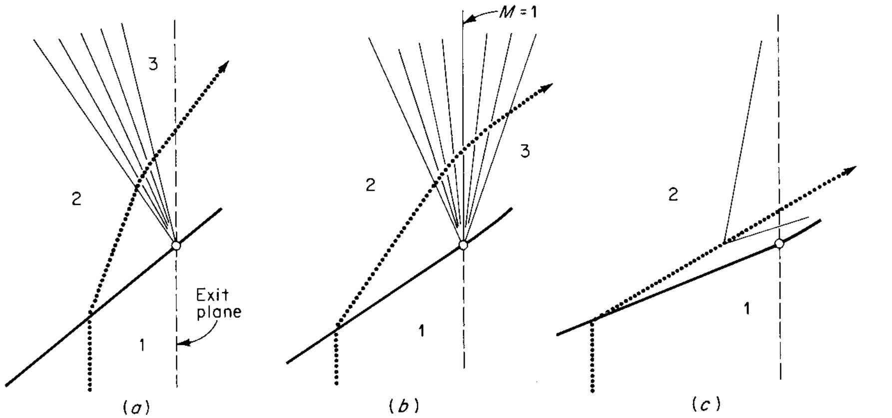

Figure 8.38
Shock incident on an open end.

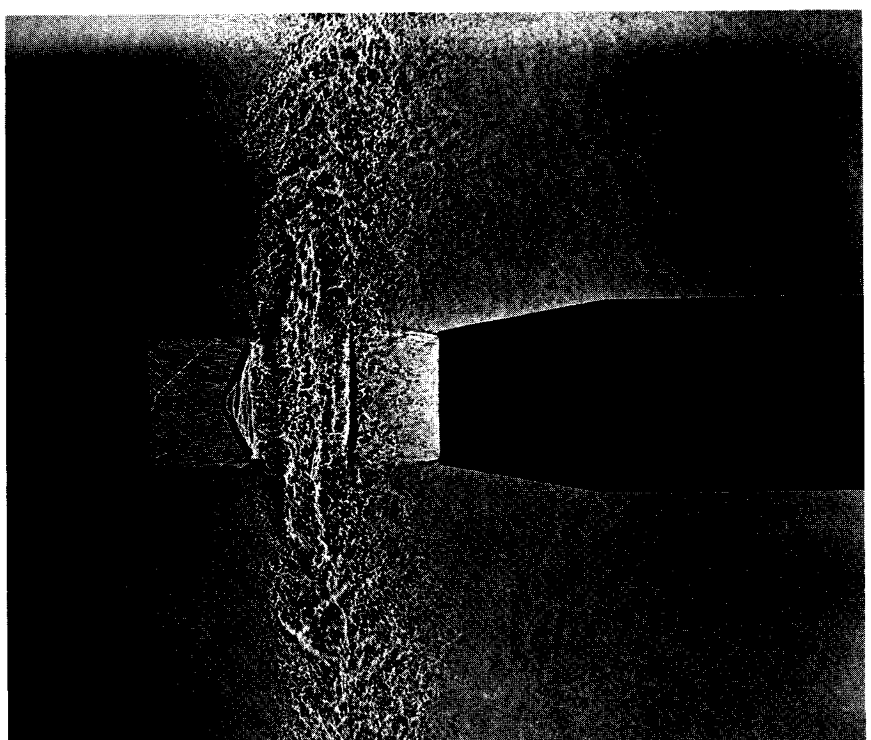

Figure 8.39
The choked jet from the right results from the passage of a shock out of the constant-area tube (unsteady flow).

That the velocity jump is the same across the shock and the reflected rarefaction fan is a consequence of the weakness of the incident shock. The flow in field 3, the outflow, is indeed subsonic since

$$M_3 = \frac{u_3}{c_3} = \frac{u_3}{c_1}\frac{c_1}{c_3} = 0.748$$

Case b. Under the same assumptions as above, and proceeding in exactly the same way, obtain

$$\frac{u_3}{c_1} = 1.389 \qquad \frac{c_3}{c_1} = 1.010$$

The flow is now supersonic in field 3, which in fact lies *outside* the tube [this can be verified by calculating the slope of the final characteristic C^- in the fan: $(u_3 - c_3)/c_1 = +0.373$, as drawn in Fig. 8.38*b*]. That part of the calculation which falls outside the tube is of course invalid. The result which is valid is that the exit flow is sonic, corresponding to the vertical characteristic standing in the mouth of the tube. The pressure P_e along this characteristic is found to be $P_e = 1.639P_1$, with expansion to pressure P_1 taking place outside the tube. For a photograph of such an expansion, see Fig. 8.39.

Case c. The shock itself is sufficiently strong to accelerate the gas to supersonic speed. From the shock tables

$$\frac{u_2}{c_1} = 1.750 \qquad \frac{c_2}{c_1} = 1.462$$

Thus $M_2 = 1.197$, and both sets of characteristics travel toward the exit plane. There can be no reflected wave, and the outflow is supersonic at state 2.

8.9 Elementary devices

The Simple Shock Tube

The shock tube is a device for producing high-temperature gas flows with relatively high Mach numbers and short time duration. It has a variety of applications in high-temperature research for physics, chemistry, and aeronautics.

In its simplest form the shock tube is a pipe closed at the ends and divided into two interior regions by a *diaphragm*. The diaphragm, which is purposely of limited strength, separates high-pressure *driver gas* from low-pressure *test gas* (driven gas). Under sufficient pressure loading the diaphragm ruptures; as an idealization it may be considered to disappear entirely at the time of rupture.