

### HW#3

#### Fanno- Rayleigh Flow

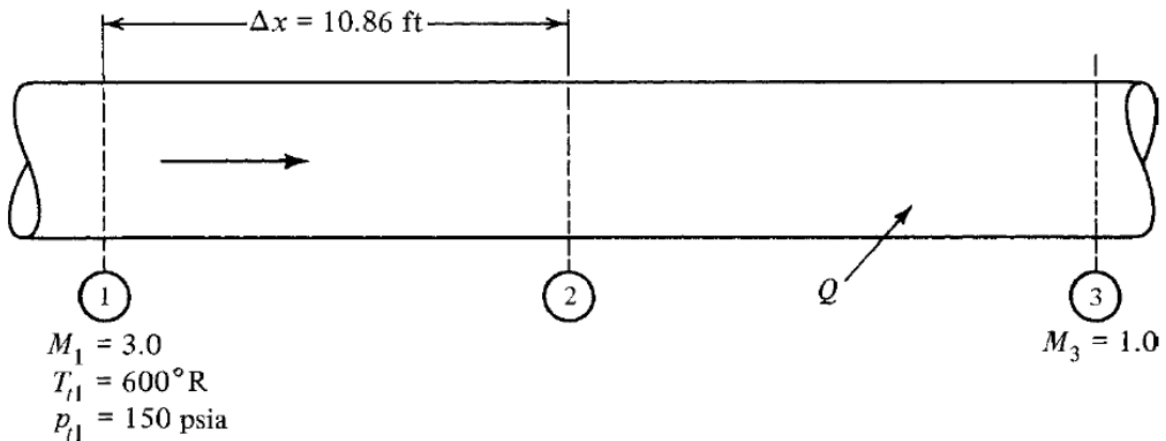
Due: 31/1/92

1- Show that for a constant-area, frictionless, steady, one-dimensional flow of a perfect gas, the maximum amount of heat that can be added to the system is given by the expression

$$\frac{q_{\max}}{c_p T_1} = \frac{(M_1^2 - 1)^2}{2M_1^2(\gamma + 1)}$$

2. Show that the maximum (static) temperature in Rayleigh flow occurs when the Mach number is  $\sqrt{1/\gamma}$

3. The 12-in.-diameter duct shown in figure below has a friction factor of 0.02 and no heat transfer from section 1 to 2. There is negligible friction from 2 to 3. Sufficient heat is added in the latter portion to just choke the flow at the exit. The fluid is air.



- (a) Draw a  $T-s$  diagram for the system, showing the complete Fanno and Rayleigh lines involved.
- (b) Determine the Mach number and stagnation conditions at section 2.
- (c) Determine the static and stagnation conditions at section 3.
- (d) How much heat was added to the flow?

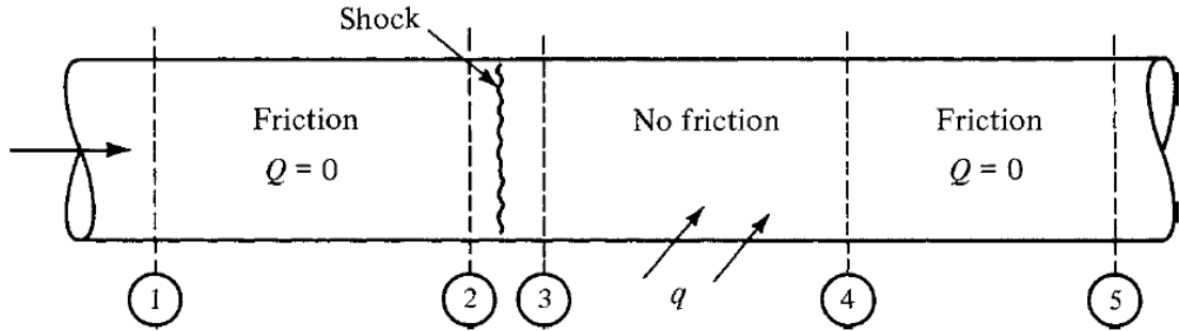
4. Recall the expression  $ptA^* = \text{const.}$

(a) State whether the following equations are true or false for the system shown in figure below.

(i)  $p_{t1}A^*_1 = p_{t3}A^*_3$

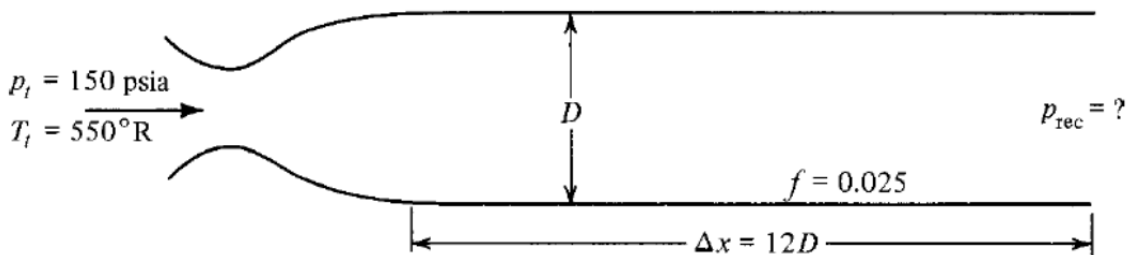
(ii)  $p_{t3}A^*_3 = p_{t5}A^*_5$

(b) Draw a  $T-s$  diagram for the system shown in figure below. Are the flows from 1 to 2 and from 4 to 5 on the same Fanno line?



5. A converging–diverging nozzle has an area ratio of 3.0. The stagnation conditions of the inlet air are 150 psia and 550°R. A constant-area duct with a length of 12 diameters is attached to the nozzle outlet. The friction factor in the duct is 0.025.

- (a) compute the receiver pressure that would place a shock
- (i) in the nozzle throat;
  - (ii) at the nozzle exit;
  - (iii) at the duct exit.
- (b) What receiver pressure would cause supersonic flow throughout the duct with no shocks within the system (or after the duct exit)?
- (c) Make a sketch showing the pressure distribution for the various operating points of parts (a) and (b).



6. show that the entropy change between two points in Rayleigh flow can be represented by the following expression if the fluid is a perfect gas:

$$\frac{s_2 - s_1}{R} = \ln \left( \frac{M_2}{M_1} \right)^{2\gamma/(\gamma-1)} \left( \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \right)^{(\gamma+1)/(\gamma-1)}$$

Introduce the \* reference condition and obtain an expression for  $(s^* - s)/R$ .