Gas Dynamics

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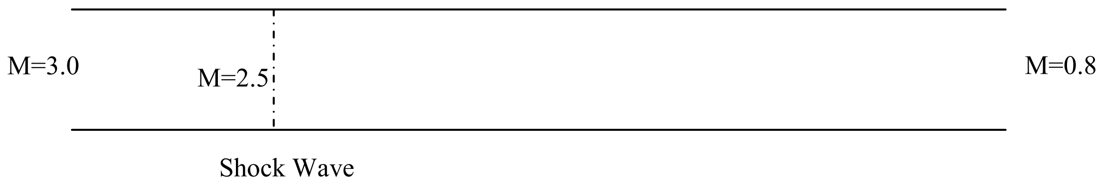
HW#3 Due Date: 9/8/1390

1- Air enters a circular duct with a diameter of d=0.61 m and friction coefficient of f=0.005. The entrance properties are M= 3.0, T= 310 K, P=70 KPa. At a station where M=2.5, a normal shock occurs. The flow continues and exit Mach number is 0.8. Take γ= 1.4,

a) Calculate the distance from the entrance of the duct to the location of the normal shock wave,
b) The total length of the duct,
c) The stagnation pressure at the duct exit,
d) The static pressure at the duct exit,
e) The entropy change between inlet and outlet,
f) Draw the H-K diagram for this process schematically.



2- Investigate the change in entropy across a shock wave where the upstream Mach number is very close to 1. Do this by making the substitution

$$M_1^2 = 1 + \epsilon$$

in the equation for the entropy change across a shock

$$S_2 - S_1 = C_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

Once the substitution is made, use the fact that ε << 1 to develop the result in a power series of ε. To do this you will need to use the binomial theorem as well as the series expansion for the logarithm. You will need to retain terms of $O(\varepsilon^3)$ in order to obtain a meaningful result. (Note: This relation is already given in your class notes)

(a) what does this result tell you about the change in entropy across a shock when the upstream Mach number is less than 1 ($\varepsilon < 0$). Show that $M_1 < 1$ corresponds to an expansion i.e. $p_2/p_1 < 1$. According to the 2nd law of thermodynamics is an expansion shock possible?

(b) Repeat the steps in part (a) for a compression shock ($\varepsilon > 0$).

(c) A compression shock with upstream Mach number close to 1 is referred to as a "weak shock". What can be said regarding the entropy produced by a weak shock. Can you think of an alternate, simpler set of equations to replace the shock jump relations in this case?

3- A converging-diverging nozzle receives air from a large reservoir, where the pressure and temperature are 200 kPa and 300 K respectively. The exit and throat cross sectional areas of the nozzle are 0.002 m^2 and 0.001 m^2 respectively. Determine

a) the maximum back pressure to choke the nozzle,

b) the back pressure for the nozzle to be perfectly expanded to the design Mach number,
c) the back pressure such that there is a normal shock wave at the exit plane
d) the ranges of back pressure for subsonic, non-isentropic, overexpansion and underexpansion flow regimes.

4- A converging-diverging nozzle is supplied air from a large reservoir. The nozzle has a throat area of 10 cm^2 and an exit area of 20 cm^2. the pressure and the temperature in the reservoir are 300 kPa and 30 C respectively.

a) Determine the ranges of back pressure over which (i) a normal shock wave appears in the nozzle, (ii) oblique shock waves form at the exit and (iii) the expansion waves form.

b) Calculate the mass flow rate through the nozzle for the case where a normal shock wave appears at the exit plane.