[redacted] **Gas Dynamics** [redacted]

1. Using the total energy form of the energy Equation
$$\rho \frac{De_\circ}{Dt} = -\nabla \cdot (p\vec{V}) + \rho\dot{q} + \rho(\vec{f} \cdot \vec{V}) \qquad \text{where} \qquad e_\circ = e + \frac{V^2}{2} \tag{1}$$
and Euler's equation
$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho\vec{f}$$
obtain the following form of the energy equation
$$\rho \frac{De}{Dt} = -p\nabla \cdot \vec{V} + \rho\dot{q} \tag{2}$$

2. By using the definition of enthalpy $h = e + pv = e + p/\rho$, the continuity equation and relation (2) obtain an alternative form of the energy equation in terms of enthalpy and pressure i.e.
$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + \rho\dot{q} \tag{3}$$

3. By using the definition of total enthalpy $h_\circ = h + V^2/2$, equations (1) and (2) derive the total enthalpy form of the energy equation which is
$$\rho \frac{Dh_\circ}{Dt} = \frac{\partial p}{\partial t} + \rho\dot{q} + \rho(\vec{f} \cdot \vec{V}) \tag{4}$$
Hint: Add eq's (1) and (3) and subtract the results from equation (2).

4. Consider the case of steady, invisid, adiabatic flow and let the body force be given by the conservation force field $\mathbf{G} = -\nabla\mathbf{\Phi}$. Start with the appropriate form of the energy equation and develop the following:
   (a) Show how one derives a very important result for the quantity $(h^2 + u^2/2 + \Phi)$.
   (b) Does this result hold if the flow is rotational flow at upstream locations?
   (c) Does this result depend on the assumption of an ideal gas or does it also apply for a real gas?
   (d) What is the appropriate statement to make concerning the case of non-steady flow, and assumption is required in order to develop a corresponding result? Give a clear and complete statement for each case, not simply a yes or no answer.

5. In chapter 3 of your text, the conservation form of the governing equations is written in terms the vector of conserved variables, $\mathbf{U}$, and the flux vectors $\mathbf{F}$ and $\mathbf{G}$. Determine the flux Jacobian $\mathbf{A}$ for a perfect gas by performing
$$\mathbf{A} \equiv \frac{\partial \mathbf{F}}{\partial \mathbf{U}} = \frac{\partial(f_1, f_2, f_3, f_4)}{\partial(u_1, u_2, u_3, u_4)}$$
Also show that the following is true:
$$\mathbf{F} = \mathbf{AU}$$
(Hint: express $\mathbf{A}$ in terms of ρ, u, v, a, where a is the sound speed for a perfect gas.)