Brief History of the Early Development of Theoretical and Experimental Fluid Dynamics

John D. Anderson Jr.
Aeronautics Division, National Air and Space Museum, Smithsonian Institution, Washington, DC, USA

Encyclopedia of Aerospace Engineering.
Edited by Richard Blockley and Wei Shyy
 ISBN: 978-0-470-68665-2

1 INTRODUCTION

As you read these words, there are millions of modern engineering devices in operation that depend in part, or in total, on the understanding of fluid dynamics – airplanes in flight, ships at sea, automobiles on the road, mechanical biomedical devices, and so on. In the modern world, we sometimes take these devices for granted. However, it is important to pause for a moment and realize that each of these machines is a miracle in modern engineering fluid dynamics wherein many diverse fundamental laws of nature are harnessed and combined in a useful fashion so as to produce a safe, efficient, and effective machine. Indeed, the sight of an airplane flying overhead typifies the laws of aerodynamics in action, and it is easy to forget that just two centuries ago, these laws were so mysterious, unknown or misunderstood as to preclude a flying machine from even lifting off the ground; let alone successfully flying through the air.

In turn, this raises the question as to just how did our intellectual understanding of fluid dynamics evolve? To find the answer, we have to reach back over millennia of intellectual thought, all the way back to ancient Greek science. However, properly addressing the history of fluid dynamics in a complete fashion requires many more pages than available in the present chapter. Several books have been written on the subject, notably those by Rouse and Ince (1957) and Tokaty (1971). An inclusive study of the history of both fluid dynamics and aerodynamics can be found in the recent book by Anderson (1997).

Instead, we will focus on a few themes and case histories that exemplify the historical evolution of fluid dynamics and provide a flavor of the intellectual thought and the human dynamics that have led to the state-of-the-art of fluid dynamics as we know it today. We will choose a chronological approach to the subject, and will marble together advancements in both theoretical and experimental fluid dynamics. Much of the following material is excerpted from the author's broader study of the subject in Anderson (1997).

2 EARLY GREEK SCIENCE: ARISTOTLE AND ARCHIMEDES

The science of fluid dynamics can trace its roots to a man born in 384 B.C. in the Ionian colony of Stagira on the Aegean Sea, and educated at Plato's Academy in Athens. Aristotle (384–322 B.C.) lived at the most intellectually fruitful time in Greek history, went to the best school, and associated with some of the most influential people. Throughout all of this, Aristotle developed a corpus of philosophy, science, ethics, and law that influenced the world for the following 2000 years.

Aristotle's scientific thoughts established two concepts that bear on the development of fluid dynamics. The first is the concept of *continuum*. He wrote that

The continuous may be defined as that which is divisible into parts which are themselves divisible to infinity, as a body which is divisible in all ways. Magnitude divisible in one direction is a line, in three directions a body. And magnitudes which are divisible in this fashion are continuous.

It is not widely appreciated that the fundamental concept of a continuum, upon which most fluid dynamic theory is based, is one of Aristotle's contributions to the science of fluid dynamics.

The second of Aristotle's contributions to aerodynamics was the idea that a moving body passing through the air or another fluid encounters some aerodynamic "resistance," He wrote:

"It is impossible to say why a body that has been set in motion in a vacuum should ever come to rest. Why, indeed, should it come to rest at one place rather than another. As a consequence, it will either necessarily stay at rest, or if in motion, will move indefinitely unless some obstacle comes into collision."

A conclusion from this reasoning is that, since bodies eventually come to rest in a fluid, there must be a *resistance* acting on the body. Today, we call this fluid dynamic *drag*.

The other Ancient Greek scientist to contribute to fluid dynamics was born in 287 BC in Syracuse, and was killed unceremoniously in 212 BC by Roman soldiers while he was drawing geometric figures in Syracuse sand. Archimedes is usually known for his concepts in fluid statics, and particularly for his vague concept of pressure in a fluid. He sensed that every point of the wetted surface area of a body in a fluid was under some force due to the fluid although the concept of "force" was not quantified during the age of Greek science. However, there was some vague, intuitive feeling about what we today technically label as force, and Archimedes realized that such force is distributed over the body surface. Archimedes stated that, in a fluid, "each part is always pressed by the whole weight of the column perpendicularly above it." This was the first statement of the principle that, in modern terms, the pressure at a point in a stationary fluid is due to the weight of the fluid above it, and hence is linearly proportional to the depth of the fluid. This is a true statement, as long as the fluid is not in motion, that is, for fluid *statics*.

However, Archimedes made a contribution to fundamental fluid dynamics as follows. Today, we fully understand that, in order to set a stagnant fluid into motion, a *difference* in pressure must be exerted across the fluid. We call this pressure difference over a unit length the *pressure gradient*. Archimedes had a vague understanding of this point when he wrote "if fluid parts are continuous and uniformly distributed, then that of them which is the least compressed is driven along by that which is more compressed." Liberally interpreted, this means that when a pressure gradient is imposed across a stagnant fluid, the fluid will start to move in the direction of decreasing pressure. The above statement by Archimedes is a clear contribution of Greek science to fluid dynamics.

3 DA VINCI'S FLUID DYNAMICS

The time-span from the death of Archimedes to the time of Leonardo da Vinci covers the zenith of the Roman Empire, its fall, the dearth of intellectual activity in Western Europe during the Dark Ages, and the surge of new thought that characterized the Renaissance. In terms of the science of aerodynamics, the seventeen centuries that separate Archimedes and Leonardo resulted in no worthwhile contributions. Although the Romans excelled in highly organized civil, military, and political activities, as well as in large engineering feats with building construction and the wide distribution of water from reservoirs to cities via aqueducts, they contributed nothing of substance to any scientific theory. Moreover, although the ancient Greek science and philosophy was kept alive for future generations by eastern Arabian cultures through the Dark Ages, no new contributions were made during this period.

This changed with the work of Leonardo da Vinci. Born in 1452 in the small Tuscan village of Vinci, near Florence, Leonardo da Vinci went on to revolutionize the worlds of art, science, and technology. He is recognized today as being in the forefront of the world's greatest intelligences.

Pertinent to this article, Leonardo had an interest in the characteristics of basic fluid flow. For example, one of the fundamental principles of modern fluid mechanics is the fact that mass is conserved; in terms of a fluid moving steadily in a tube, this means that the mass flow (e.g., the number of pounds per second) passing through any cross section of the tube is the same. For an incompressible flow (flow of a fluid, or low-speed flow of a gas), this principle leads to a basic relation that

$$AV = \text{constant} \tag{1}$$

where A is the cross-sectional area of the duct at any location, and V is the velocity of the fluid at that same location. This relation is called the *continuity equation*, and it states that in moving from one location in the duct to another where the area is smaller, the velocity becomes larger in just the right amount that the product of A times V remains the same. Leonardo observed and recorded this effect in regard to the flow of water in rivers, where, in those locations where the river becomes constricted, the water velocity increases. Moreover, he quantified this observation in the following statement that refers to water flow through a passage where the depth *mn* changes to a smaller depth, *ab*, smaller by factor of 4.

"Each movement of water of equal surface width will run the swifter the smaller of the depth. . .and this motion will be of this quality: I say that in *mn* the water has more rapid movement than in *ab*, and as many times more as *mn* enters into *ab*; it enters 4 times, the motion will therefore be 4 times as rapid in *mn* as in *ab*."

Here we have, for the first time in history, a quantitative statement of the special form of the continuity equation that holds for low-speed flow.

In addition to this quantitative contribution, Leonardo, being a consummate observer of nature, made many sketches of various flowfields. A particularly graphic example is shown in Figure 1, found in the Codex Atlanticus. Here we see the vortex structure of the flow around a flat plate. At the top, the plate is perpendicular to the flow, and Leonardo accurately sketches the recirculating, separated flow at the back of the plate, along with the extensive wake that trails downstream. At the bottom, the plate is aligned with the flow, and we see the vortex that is created at the junction of the plate surface and the water surface, as well as the bow wave that propagates at an angle away from the plate surface. These sketches by Leonardo are virtually identical to photographs of such flows

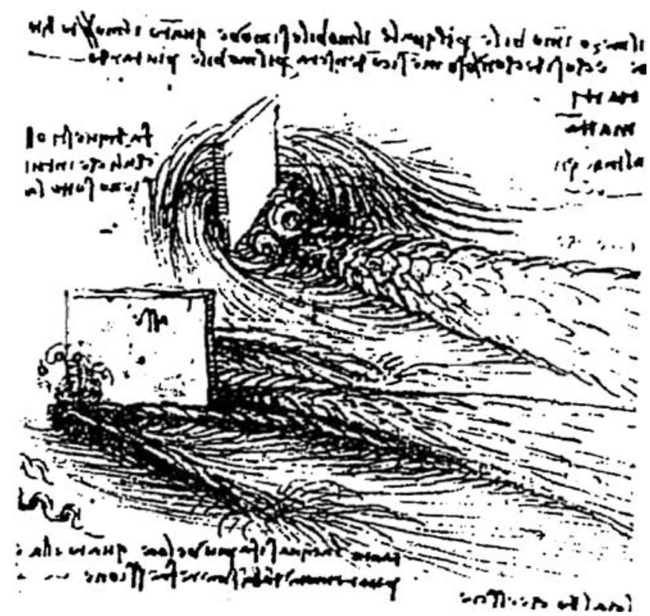

Figure 1. Sketches by da Vinci showing complex flow fields over objects in a flowing stream.

that can be taken in any modern fluid dynamic laboratory, and they demonstrate the detail to which Leonardo observed various flow patterns.

In modern fluid dynamics and aerodynamics, the wind tunnel is an absolutely essential laboratory device. Although we take for granted today that the relative flow over a stationary body mounted in a wind tunnel is the same as the relative flow over the same body moving through a stationary fluid, we have Leonardo to thank for being the first to state this fact. His statement of what we can call today the "wind tunnel principle" can be found in two different parts of the *Codex Atlanticus*. Leonardo made the following statements: "As it is to move the object against the motionless air so it is to move the air against the motionless object," and "The same force as is made by the thing against air, is made by air against the thing." Therefore, the basic principle that allows us to make wind tunnel measurements and apply them to atmospheric flight was first conceived by Leonardo 380 years before the invention of the first wind tunnel.

4 THE VELOCITY-SQUARED LAW

We now address what is perhaps the most important breakthrough in experimental fluid dynamics in the 17th century. Put yourself in the shoes of a self-styled natural philosopher in the Middle Ages. In thinking about the question of how the force on an object immersed in a moving fluid varies with the velocity of the fluid, intuition is most likely to tell you that, when the velocity doubles, the force doubles. That is, you are inclined to feel that force is directly proportional to velocity. This seems "logical," although there is (up to the 17th century) no proper experimental evidence or theoretical analysis to say one way or another. Like so much of ancient science, this feeling was based simply on the image of geometric perfection in nature, and what could be more "perfect" than the force doubling when the velocity doubles. Indeed, both Leonardo and Galileo – two of the greatest minds in history – held this belief. Up to the middle of the 17th century, the prevailing thought was the incorrect notion that force was directly proportional to the flow velocity.

However, within the space of 17 years at the end of the seventeenth century, this situation changed dramatically. Between 1673 and 1690, two independent sets of experiments due to Edme Mariotte (1620–1684) in France and Christian Huygens (1629–1695) in Holland, along with the theoretical fundamentals published by Isaac Newton (1642–1727) in England, clearly established that the force on an object varies as the square of the flow velocity, that is, if the velocity doubles, the force goes up by a factor of four. In comparison to the previous centuries of halting, minimal progress in fluid dynamics, the rather sudden realization of the velocity-squared law for aerodynamic forces represents the first major scientific breakthrough in the historical evolution of the subject. Let us examine this breakthrough more closely, as well as the men who made it possible.

Credit for the origin of the velocity-squared law rests with Edme Mariotte, who first published it in the year 1673. To gain an appreciation for the circumstances surrounding this development, let us consider Mariotte's background. He lived in absolute obscurity for about the first 40 yr of his life. There is even controversy as to where and when he was born. There is a claim that he was born in Dijon, France, in 1620, but there are no documents to verify this, let alone to pinpoint an exact birth date. We have no evidence concerning his personal life, his education, or his vocation until 1666, when very suddenly he was made a charter member of the newly formed Paris Academy of Sciences. Most likely, Mariotte was self-taught in the sciences. He came to the attention of the Academy through his pioneering theory that sap circulated through plants in a manner analogous to blood circulating through animals. Controversial at that time, his theory was confirmed within four years by numerous experimental investigators. It is known that he was residing in Dijon at the time of his appointment to the Academy. Mariotte quickly proved to be an active member and contributor to the Academy. His areas of work were diverse; he was interested in experimental physics, hydraulics, optics, plant physiology, surveying, and general scientific and mathematical methodology. Mariotte is credited as the first in France to develop experimental science, transferring to that country the same interest in experiments that grew during the Italian renaissance with the work of Leonardo and Galileo. Indeed, Mariotte was a gifted experimenter who took pains to try to link existing theory to experiment – a novel thought in that day. The Academy was essentially Mariotte's later life; he remained in Paris until his death on 12 May 12 1684.

The particular work of Mariotte of interest to our discussion was conducted in the period before 1673. He was particularly interested in the forces produced by various bodies impacting on other bodies or surfaces. One of these "bodies" was a fluid; Mariotte examined and measured the force created by a moving fluid impacting on a flat surface. The device he used for these experiments was a beam dynamometer wherein a stream of water impinges on one end of the beam, and the force exerted by this stream is balanced and measured by a weight on the other end of the beam. The water jet emanates from the bottom of a filled vertical tube, and its velocity is known from Torricelli's law as a function of the height of the column of water in the tube. From the results obtained with this experimental apparatus, Mariotte was able to prove that the force of impact of the water on the beam varied as the square of the flow velocity. He presented these results in a paper read to the Paris Academy of Science in 1673, entitled "Traité de la Percussion ou Choc des Corps," – the first time in history that the velocity-squared law was published. For this work, Edme Mariotte deserves the credit for the first major advancement toward the understanding of velocity effects on aerodynamic force.

As a final note on Mariotte, the esteem in which he was held by some of his colleagues is reflected by the words of J.B. du Hamel, who said after Mariotte's death in 1864,

> "The mind of this man was highly capable of all learning, and the works published by him attest to the highest erudition. In 1667, on the strength of a singular doctrine, he was elected to the Academy. In him, sharp inventiveness always shone forth combined with the industry to carry through, as the works referred to in the course of this treatise will testify. His cleverness in the design of experiments was almost incredible, and he carried them out with minimal expense."

However, there was at least one colleague who was not so happy with Mariotte, and who represents another side of the historical proprietorship of the velocity-squared law. This man was Christian Huygens (1629–1695). Huygens' background is better known than that of Mariotte. Christian Huygens was born on April 14, 1629, in the Hague, The Netherlands, to a family prominent in Dutch society. His grandfather served William the Silent and Prince Maurice as secretary. His father, Constantine, was secretary to Prince Frederick Henry. Indeed, several members of the family were diplomats under the reign of the Orange family in Holland. Christian was well educated; he was tutored by his father until the age of 16, after which he studied law and mathematics at the University of Leiden. Devoting himself to physics and mathematics, Huygens made substantial contributions, including improvements in existing methodology, developing new techniques in optics, and inventing the pendulum clock. Even today, all textbooks on basic physics discuss Huygens' law of optics. For his accomplishments, Huygens was made a charter member of the Paris Academy of Science in 1666 – the same year as Mariotte. Huygens moved to Paris in order to more closely participate in the activities of the Academy; he lived in Paris until 1681. During this life, Huygens was recognized as Europe's greatest mathematician. However, he was a somewhat solitary person who did not attract a following of young students. Moreover, he was reluctant to publish, mainly because of his inordinately high personal standards. For these two reasons, Huygens' work did not greatly influence the scientists of the next century; indeed, he became relatively unknown during the 18th century.

In 1668, Huygens began to study the fall of projectiles in resisting media. Following Leonardo and Galileo, he started out with the belief that resistance (drag) was proportional to velocity. However, within one year his analysis of the experimental data convinced him that resistance was proportional to the square of the velocity. This was four years before Mariotte published the same result in 1673; however, Huygens delayed until 1690 in publishing his data and conclusions. This somewhat complicates the question as to whom should the velocity-squared law be attributed. The picture is further blurred by Huygens himself, who accused Mariotte of plagiarism; however, Huygens levied this charge after Mariotte's death in 1684. Huygens stated that "Mariotte took everything from me." In regard to Mariotte's paper in 1673, Huygens complains that "he should have mentioned me. I told him that one day, and he could not respond."

In the present author's opinion, here is a classic situation that frequently occurs in scientific and engineering circles even in modern times. We have a learned society – the Paris Academy of Sciences – the members of which frequently gathered to discuss their experiments, theories, and general feelings about the natural world. Ideas and preliminary results were shared and critiqued in a collegial atmosphere. Mariotte and Huygens were colleagues, and from Huygens own words above, they clearly discussed and shared thoughts. In such an atmosphere, the exact credit for the origin of new ideas is sometimes not clear; ideas frequently evolve as a result of discussion among groups. What is clear is this. Mariotte published the velocity-squared law in a paper given to the Academy in 1673; Huygens published the same conclusion 17 years later. Moreover, in 1673 Huygens critiqued Mariotte's paper, and said nothing about plagiarism or not being referenced. Why did he wait until after Mariotte's death 11 years later to make such charges? This author has no definite answer to this question. However, using the written scientific literature as the measure of proprietorship, Mariotte is clearly the first person to publish the velocity-squared law. Taken in conjunction with Huygens' silence at the time of this publication, we have to conclude that Mariotte deserves first credit for this law. However, it is quite clear that Huygens' experiments, which were carried out before Mariotte's publication, also proved the velocity-squared law. Of course, of great importance to the development of fluid dynamics is simply the fact that, by the end of the 17th century, we have direct experimental proof from two independent investigations that fluid dynamic force varies as the square of the velocity. Of even greater importance is that, at the same time, the same law was derived theoretically on the basis of the rational, mathematical laws of mechanics advanced by Newton in his Principia, published in 1687.

5 NEWTON AND THE SINE-SQUARED LAW

It is fitting that the end of the 17th century saw the natural fruition of experimental work such as that by Mariotte and Huygens in the development of a rational mathematical theory by Isaac Newton (1642–1727). Newton's contributions to physics and mathematics were pivotal. The publication of his "Philosophiae Naturalis Principia Mathematica" – widely known as "the Principia" in 1687 represented the first complete, rational, theoretical approach to the study of mechanical phenomena.

Newton was born on 25 December 1642, in the hamlet of Woolsthorpe-by-Colsterworth near the English town of Grantham. He was raised by his mother; his father had died five months before his birth. He showed an interest in mechanical diagrams, which he scratched on the walls and window edges of his house in Woolsthorpe. With the encouragement of an uncle, Newton entered Trinity College at Cambridge in 1661 and received his B.A. degree in 1665. For the next two years he retreated to the country in Lincolnshire to avoid the plague that was running rampant in Europe and had closed the University. It was during that two-year period that he conceived many of his basic ideas on mathematics, optics, and mechanics that were later to appear in print. Newton said of those two years that "I was in the prime of my age of invention and minded mathematics and philosophy more than at any time since." In 1667, Newton returned to Cambridge and became a minor fellow at Trinity. He earned an M.A. degree in 1668 and was appointed Lucasian professor in 1669. Newton remained at Cambridge for the next 27 years.

Newton's contributions to fluid dynamics appear in Book II of the *Principia*, subtitled "The Motions of Bodies (in

Resisting Mediums).” Book II deals exclusively with fluid dynamics and hydrostatics. During the last part of the seventeenth century, practical interest in fluid dynamics was driven by problems in naval architecture, particularly the need to understand and predict the drag on a ship’s hull, an important concern for a country that was ruling large portions of the world through the superior performance of its powerful navy. Newton’s interest in fluid mechanics may have derived partly from such a practical problem, but he had a much more compelling reason for calculating the resistance of a body moving through a fluid. There was a prevailing theory, advanced by Rene Descartes, that interplanetary space was filled with matter that moved in vortex-like motions around the planets. However, astronomical observations, such as the definitive work of Johannes Kepler in his *Rudolphine Tables*, published in 1627, indicated that the motions of the heavenly bodies through space were not dissipated, but rather that those bodies executed regular, repeatable patterns. The only explanation that the bodies were moving through space filled with a continuous medium as Descartes had theorized would be for the aerodynamic drag on each body to be zero. The central purpose of Newton’s studies in fluid mechanics was to prove that there was a finite drag on a body (including the heavenly bodies) moving through a continuous medium. If that could be shown to be true, then the theory of Descartes would be disproved. Indeed, in Proposition 23 of the *Principia*, Newton calculated *finite* resistance on bodies moving through a fluid and showed that such resistances were “in a ratio compounded of the squared ratio of their velocities, and the squared ratio of their diameters, and the simple ratio of the density of the parts of the system.” That is, Newton *discussed* the velocity-squared law, while at the same time showing that resistance varies with the cross-sectional area of the body (the “squared ratio of their diameters”) and the first power of the density (the “simple ratio of the density”). In so doing, Newton presented the first theoretical derivation of the essence of the drag equation

$$D \propto \rho \, \mathrm{S} \, \mathrm{V}^2 \tag{2}$$

However, in Newton’s mind, his contribution was simply to refute the theory of Descartes. That was stated specifically by Newton in the scholium accompanying Proposition 40, dealing with experimental measurements of the resistance of a sphere moving through a continuous medium. Because such spheres had been shown both theoretically and experimentally to exhibit *finite* resistances while moving through a fluid, Newton reasoned that “the celestial spaces, through which the globes of the planets and comets are continually passing towards all parts, with the utmost freedom, and without the least sensible diminution of their motion, must be utterly void of any corporeal fluid, excepting, perhaps, some extremely rare vapors and the rays of light.” For Newton, that was the crowning accomplishment from his study of fluid dynamics.

In regard to aerodynamics, Newton’s work in Book II of the *Principia* contributed a second fundamental finding, namely, a relationship for the shear stress at any point in a fluid in terms of the velocity gradient existing at that same point. Newton advanced the following hypothesis: “The resistance arising from the want of lubricity in the part of a fluid is, other things being equal, proportional to the velocity with which the parts of the fluid are separated from one another.” In modern terms, the “want of lubricity” is the action of friction in the fluid, namely, the shear stress τ. The “velocity with which the parts of the fluid are separated from one another” is the rate of strain experienced by a fluid element in the flow, which in turn can be mathematically represented by the velocity gradient, $\mathrm{d}V/\mathrm{d}n$. A mathematical statement of Newton’s hypothesis is simply

$$\tau \propto \mathrm{d}V/\mathrm{d}n \tag{3}$$

With the proportionality constant defined at the coefficient of viscosity, μ, that becomes

$$\tau = \mu(\mathrm{d}V/\mathrm{d}n) \tag{4}$$

This equation is called the Newtonian shear-stress law, and all fluids that obey the law are called “Newtonian fluids.” Virtually all gases, including air, are Newtonian fluids. Hence the Newtonian stress law, as first hypothesized in the *Principia* represented a major contribution to the state of the art of fluid dynamics at the end of the seventeenth century.

In an indirect sense, Isaac Newton was responsible for the first technical contribution toward the analysis of angle-of-incidence (angle of attack) effects on aerodynamic force. Proposition 34 in Book II of the *Principia* is a proof that the resistance of a sphere moving through a fluid is half that of a circular cylinder of equal radius with its axis oriented in the direction of its motion. The fluid itself is postulated as a collection of individual particles in rectilinear motion that impact directly on the surface of the body, subsequently giving up their components of momentum normal to the surface, and then traveling downstream tangentially along the body surface. That fluid model was simply a hypothesis on the part of Newton; it did not accurately model the action of a real fluid, as Newton readily acknowledged. However, consistent with that mathematical model, buried deep in the proof of Proposition 34 is the result that the impact force exerted by the fluid on a segment of a curved surface is proportional to $\sin^2\theta$, where θ is the angle between a local tangent to the surface and the free-stream direction. That result, when applied to a

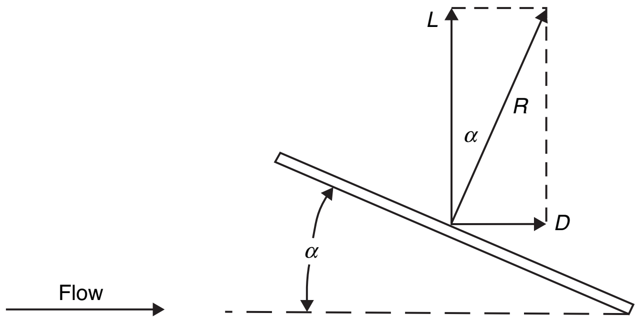

Figure 2. Aerodynamic force on a flat plate at an angle of attack a.

flat surface (e.g., a flat plate) oriented at an angle of attack α to the free stream (Figure 2), gives the resultant aerodynamic force R on the plate:

$$R = \rho V^2 S \sin^2\alpha \tag{5}$$

where S is the planform area of the plate. This equation is called the *Newtonian sine-squared law*.

The Newtonian sine-squared law is a simple relationship for the calculation of the fluid dynamic force on a surface, and for that reason other experimenters were quick to use it. However, the accuracy of the Newtonian sine-squared law soon came into question in the 18th century. For example, in 1777 the great French scientist and mathematician Jean le Rond d'Alembert participated in a series of experimental measurements of the drag on ships' hulls. For the most part, calculations from the sine-squared law did not agree with the experimental data. However, other researchers continued to use the sine-squared law for another century.

The Newtonian sine-squared law was used by various would-be flying machine inventors in the 19th century for the prediction of lift L and drag D. In Figure 2, the resultant force R is resolved into two mutually perpendicular forces: lift L perpendicular to V and drag D parallel to V. Hence,

$$\begin{aligned} L &= R\cos\alpha = \rho V^2 S \sin^2\alpha \cos\alpha \\ D &= R\sin\alpha = \rho V^2 S \sin^3\alpha \end{aligned} \tag{6}$$

and, the lift-to-drag ratio L/D is

$$\frac{L}{D} = \cot\alpha \tag{7}$$

Examining the foregoing equation for lift, we note that for a flying machine at a given velocity with a given wing area S, the sine-squared law predicts very small lift at small angles of attack. However, for steady level flight, the lift must equal the weight. If we were to accept the Newtonian sine-squared law as correct, then we would have only two options to counter the small value of $\sin^2\alpha$ and to increase the lift so that it would equal the weight of the flying machine:

1. Increase the wing area S. That would lead to enormous wing areas, which would make the flying machine totally impractical.
2. Increase the angle of attack α . Unfortunately that would lead to greater drag because D varies as $\sin^3\alpha$, that is, the drag increases faster than the lift as α is increased, putting a greater demand on the power plant. The lift-to-drag ratio would decrease dramatically. Because L/D is a measure of aerodynamic efficiency, flying at large angles of attack would be undesirable, to say the least.

Indeed, Anderson (2002) shows that if the Wright brothers had used the Newtonian sine-squared law to design the 1903 Wright Flyer (they did not), the wing area would have been a whopping 23 448 ft^2 – an impossibly large area for a flying machine at that time – in comparison to the actual wing area of 510 ft^2. If the Wrights had based their design on the sine-squared law, they would have quit their efforts immediately. Indeed, throughout the nineteenth century, the Newtonian sine-squared law was misused by many naysayers to "prove" that heavier-than-air powered flight was not possible.

Ironically, the Newtonian sine-squared law has had a rebirth in modern aerodynamics, namely, for the prediction of pressure distributions on the surfaces of hypersonic vehicles. The physical nature of hypersonic flow, where the bow shock wave lies very close to the vehicle surface, closely approximates the fluid model used by Newton – a stream of particles in rectilinear motion colliding with the surface and then moving tangentially over the surface. Hence, the sine-squared law leads to reasonable predictions for the pressure distributions over blunt-nosed hypersonic vehicles, an application that Newton could not have foreseen.

6 DANIEL BERNOULLI AND THE PRESSURE-VELOCITY CONCEPT

The fundamental advances in fluid dynamics that occurred in the 18th century began with the work of Daniel Bernoulli (1700–1782). Newtonian mechanics had unlocked the door to modern hydrodynamics, but the door was still closed at the beginning of the century. Daniel Bernoulli was the first to open this door, albeit just by a crack; Euler and others who followed flung the door wide open.

Daniel Bernoulli was born in Groningen, The Netherlands on February 8, 1700. His father, Johann, was a professor at Groningen but returned to Basel, Switzerland, in 1705, to occupy the Chair of Mathematics that had been vacated by the death of Jacob Bernoulli. At the University of Basel, Daniel obtained a master's degree in 1716 in philosophy and logic. He went on to study medicine in Basel, Heidelberg, and Strasbourg, obtaining his Ph.D. in anatomy and botany in 1721. During these studies, he maintained an active interest in mathematics. He followed this interest by moving briefly to Venice, where he published an important work entitled "Exercitationes quaedam Mathematicae" in 1724. This earned him much attention and resulted in his winning the prize awarded by the Paris Academy – the first of 10 he was eventually to receive. In 1725, Daniel moved to St. Petersburg, Russia, to join the academy. The St. Petersburg Academy had gained a substantial reputation for scholarship and intellectual accomplishment at that time. During the next eight years, Bernoulli experienced his most creative period. While at St. Petersburg, he wrote his famous book *Hydrodynamica*, completed in 1734, but not published until 1738. In 1733, Daniel returned to Basel to occupy the Chair of Anatomy and Botany, and in 1750 moved to the Chair of Physics created exclusively for him. He continued to write, give very popular and well-attended lectures in physics, and make contributions to mathematics and physics until his death in Basel on March 17, 1782.

Daniel Bernoulli was famous in his own time. He was a member of virtually all the existing learned societies and academies, such as Bologna, St. Petersburg, Berlin, Paris, London, Bern, Turin, Zurich, and Mannheim. His importance to fluid dynamics is centered on his 1738 book, *Hydrodynamica* (with this book, Daniel introduced the term "hydrodynamics" to literature). In this book, he ranged over such topics as jet propulsion, manometers, and flow in pipes. Of most importance, however, he attempted to find a relationship between the variation of pressure with velocity in a fluid flow. He used Newtonian mechanics, along with the concept of "vis viva" or "living force" introduced by Leibniz in 1695. This was actually an energy concept; "vis viva" was defined by Leibniz as the product of mass times velocity squared, mV^2; today, we recognize this as twice the kinetic energy of a moving object of mass m. Also, Bernoulli treated pressure in terms of the height of a fluid, much as Archimedes had done 20 centuries previously; the concept that pressure is a point property that can vary from one point to another in a flow cannot be found in Bernoulli's work.

Let us critically examine Bernoulli's contribution to fluid dynamics. In modern fluid dynamics there exists the "Bernoulli principle," which simply states that in a flowing fluid, as the velocity increases, the pressure decreases. This is an absolute fact that is frequently used to explain the generation of lift on an airplane wing; as the flow speeds up while moving over the top surface of the wing, the pressure decreases. This lower pressure on the top surface in combination with a higher pressure on the bottom surface generates lift. A quantitative statement of Bernoulli principle is Bernoulli's equation, written as follows. If points 1 and 2 are two different points in a fluid flow, then

$$p_1 + 1/2\rho V_1^2 = p_2 + 1/2\rho V_2^2 \tag{8}$$

This is the famous Bernoulli equation – perhaps the most famous equation in all of fluid dynamics. Examining this equation, clearly if V_2 is larger than V_1, then p_2 is smaller than p_1; that is, as V increases, p decreases. Question: How much of this did Bernoulli ever state? The answer is, not much. In his book *Hydrodynamica*, which is the central reference used by all subsequent investigators for his contributions, Bernoulli did attempt to derive the relation between pressure and velocity. Using the concept of "vis viva," Bernoulli applied an energy conservation principle to the sketch shown in Figure 3; this is a copy of his original illustration for *Hydrodynamica*. Here we see a large tank, ABGC, filled with water, to which has been attached a horizontal pipe, EFDG. The end of the pipe is partially closed; it contains a small orifice through which the water escapes. Stating that the sum of the potential and kinetic energies of the fluid in a pipe is constant (an incorrect statement, because in a flowing fluid there is work done by the pressure in addition to the existence of kinetic and potential energies – such "flow work" was not understood by Bernoulli), he obtained the following differential equation for the change in velocity, dV, over a small

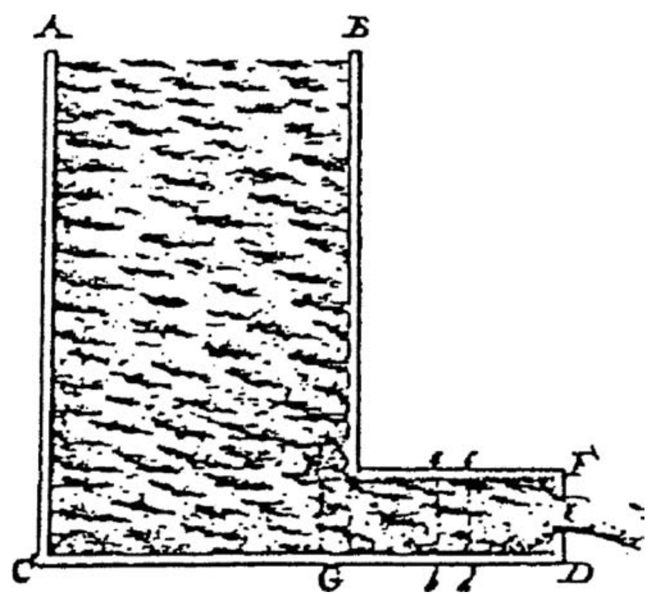

Figure 3. Sketch from Bernoulli's Hydrodynamica showing water flowing from a tank.

distance, dx.

$$\frac{V\,\mathrm{d}V}{\mathrm{d}x} = \frac{a - V^2}{2c} \tag{9}$$

where a is the height of the water in the tank, and c is the length of the horizontal pipe. The above equation is a far cry from the "Bernoulli equation" we use today. However, Bernoulli went on to interpret the term $V\mathrm{d}V/\mathrm{d}x$ as the pressure, which allows us to interpret the relation in *Hydrodynamica* as the form

$$p = \frac{a - V^2}{2c} \tag{10}$$

Since a and c are constants, this relation says qualitatively that as velocity increases pressure decreases.

From this, we are led to conclude the following:

1. The principle that pressure decreases as velocity increases is indeed presented in Bernoulli's book, albeit in a slightly obscure form. Hence, it is clearly justified to call this the "Bernoulli principle," as is done today. However, it is interesting to note that nowhere in his book does Bernoulli emphasize the importance of this principle, showing a certain lack of appreciation of its significance.
2. Bernoulli's equation does not appear in his book, nor elsewhere in his work. It is quite clear that Bernoulli never derived nor used Bernoulli's equation.

This is not to diminish Bernoulli's contributions to fluid dynamics. His work was used as a starting point by other investigators in the 18th century. He was the first to examine the relation between pressure and velocity in a flow using the new scientific principles of the 18th century. As far as this author can ascertain, he was the first to use the elements of calculus to analyze a fluid flow, as illustrated in the differential equation shown above, obtained from his *Hydrodynamica*. His work inspired the work of other investigators, including that of Euler, d'Alembert, and Lagrange.

7 HENRI PITOT AND THE INVENTION OF THE PITOT TUBE

A major advancement in experimental fluid dynamics occurred on November 12, 1732, at the Royal Academy of Sciences in Paris. On this day, Henri Pitot announced to the Academy a new invention by which he could directly measure the local flow velocity at a point in fluid. Later called the Pitot tube, this device has become the most commonplace instrument in modern 20th century fluid dynamic laboratories. Because of its importance, let us examine the historical details surrounding its development.

For Pitot himself, the invention of the Pitot tube was just one event in a reasonably productive life. Born in Aramon, France on May 3, 1695 of reasonably educated parents, Pitot's youth was undistinguished; indeed, he demonstrated an intense dislike of academic studies. While serving a brief time in the military, Pitot was motivated by a geometry text published in a Grenoble bookstore, and subsequently spent three years at home studying mathematics and astronomy. In 1718, Pitot moved to Paris, and by 1723 had become an assistant in the chemistry laboratory of the Academy of Sciences. It was to this group that he delivered, on November 12, 1732, his announcement of his new device for measuring flow velocity – the Pitot tube.

His invention of the Pitot tube was motivated by his dissatisfaction with the existing technique of measuring the flow velocity of water, which was to observe the speed of a floating object on the surface of the water. So he devised an instrument consisting of two tubes; one was simply a straight tube open at one end that was inserted vertically into the water (to measure the static pressure) and the other was a tube with one end bent at right angles with the open end facing directly into the flow (to measure total pressure) – namely, the Pitot tube. In 1732, between two piers of a bridge over the Seine River in Paris, he used this instrument to measure the flow velocity of the river at different depths within the river. In this presentation to the Academy later that year, Pitot presented his results, which had importance beyond the Pitot tube itself. Contemporary theory, based on the experience of some Italian engineers, held that the flow velocity at a given depth in a river was proportional to the mass above it; hence the velocity was thought to increase with depth. Pitot reported the stunning (and correct) results, measured with his instrument, that in reality the flow velocity decreased as the depth increased. Hence, Pitot introduced his new invention with style. Later, in 1740, he accepted an invitation from the Estates General of Languedoc to supervise the draining of swamps in the province, which then led to his becoming director of public works of the province as well as superintendent of the Canal du Languedoc. In his old age, Pitot retired to his birthplace, and died at Aramon on December 27, 1771.

The development of the Pitot tube in 1732 was a substantial contribution to experimental fluid dynamics. However, in 1732, Henri Pitot did not have the benefit of Bernoulli's equation, which was obtained by Euler 20 years later. Pitot's reasoning for the operation of this tube was purely intuitive, and he was able to correlate by empirical means the flow velocity corresponding to the measured difference between the stagnation pressure as measured by his Pitot tube, and the flow static pressure, as measured by a straight tube inserted vertically in the fluid, with its open face tube parallel to the flow. As discussed by Anderson (2008), the proper application of Bernoulli's equation to extract the velocity from the Pitot measurement of stagnation pressure was not presented until 1913. In that year, John Airey at the University of Michigan published an exhaustive experimental behavior of Pitot tubes, and presented a rational theory for their operation based on Bernoulli's equation. Invented in the early part of the eighteenth century, the Pitot tube required two centuries before it was properly incorporated into fluid dynamics as a viable experimental tool.

8 THE HIGH NOON OF EIGHTEENTH CENTURY FLUID DYNAMICS – LEONHARD EULER AND THE GOVERNING EQUATIONS OF INVISCID FLUID MOTION

Today, in the modern world of twenty-first century fluid dynamics, at the very instant that you are reading this page, there are literally thousands of fluid dynamicists who are solving the governing equations of fluid motion for an inviscid flow. Such inviscid flows – flows without friction – adequately describe many aspects of practical fluid dynamic problems as long as friction is not being considered. These solutions may involve closed-form theoretical mathematics, or more likely today may involve direct numerical solutions on a high-speed digital computer. However, the governing equations that are being solved in such a "high-tech" fashion are themselves over 200 years old; they are called the Euler equations. The development of the Euler equations represents a contribution to fluid dynamics of a magnitude much greater than any other we have discussed so far in this chapter. They represent, for all practical purposes, the true beginning of theoretical fluid dynamics. These equations were first developed by Leonhard Euler; for this reason Euler is frequently credited as being the "founder of fluid mechanics." This is somewhat of an overstatement, because as is almost always the case in physical science, Euler benefitted from earlier work, especially that of d'Alembert. On the other hand, Euler is a giant in the history of fluid dynamics, and his contributions bordered on the revolutionary rather than the evolutionary side. For these reasons, let us first take a look at Euler, the man.

Leonhard Euler was born on 15 April 1707 in Basel, Switzerland. His father was a Protestant minister who enjoyed mathematics as a pastime. Therefore, Euler grew up in a family atmosphere that encouraged intellectual activity. At the age of 13, Euler entered the University of Basel, which at that time had about 100 students and 19 professors. One of those professors was Johann Bernoulli who tutored Euler in mathematics. Three years later, Euler received his master's degree in philosophy. It is interesting that three of the people most responsible for the early development of theoretical fluid dynamics – Johann, Daniel Bernoulli, and Euler – lived in the same town of Basel, were associated with the same university, and were contemporaries. Indeed, Euler and the Bernoullis were close and respected friends – so much so that when Daniel Bernoulli moved to teach and study at the St. Petersburg Academy in 1725, he was able to convince the academy to hire Euler as well. At this invitation, Euler left Basel for Russia, he never returned to Switzerland, although he remained a Swiss citizen throughout his life.

Euler's interaction with Daniel Bernoulli in the development of fluid mechanics grew strong during these years at St. Petersburg. It was here that Euler conceived of pressure as a point property that can vary from point to point throughout a fluid, and obtained a differential equation relating pressure and velocity. In turn, Euler integrated the differential equation to obtain, for the first time in history, Bernoulli's equation in the form we use today. Hence we see that Bernoulli's equation really is a misnomer; credit for it is legitimately shared by Euler.

When Daniel Bernoulli returned to Basel in 1733, Euler succeeded him at St. Petersburg as a professor of physics. Euler was a dynamic and prolific man; by 1741 he had prepared 90 papers for publication and written the two-volume book *Mechanica*. The atmosphere surrounding St. Petersburg was conducive to such achievement. Euler wrote in 1749, "I and all others who had the good fortune to be for some time with the Russian Imperial Academy cannot but acknowledge that we owe everything which we are and possess to the favorable conditions which we had there."

However, in 1740, political unrest in St. Petersburg caused Euler to leave for the Berlin Society of Sciences, at that time just formed by Frederick the Great. Euler lived in Berlin for the next 25 years, where he transformed the society into a major academy. In Berlin, Euler continued his dynamic mode of working, preparing at least 380 papers for publication. Here as a competitor with d'Alembert, Euler formulated the basis for mathematical physics.

In 1766, after a major disagreement with Frederick the Great over some financial aspects of the academy, Euler moved back to St. Petersburg. The second period of his life in Russia became one of physical suffering. In that same year, he became blind in one eye after a short illness. An operation in 1771 resulted in restoration of his sight, but only for a few days. He did not take proper precautions after the operation, and within a few days he was completely blind. However, with the help of others, he continued his work. His mind was as sharp as ever, and his spirit did not diminish. His literary output even increased – about half of his total papers were written after 1765!

On 8 September 1783 Euler conducted business as usual – giving a mathematics lesson, making calculations of the motion of balloons, and discussing with friends the planet of Uranus, which had recently been discovered. At about 5 pm, he suffered a brain hemorrhage. His only words before losing consciousness were "I am dying." By 11 pm, one of the greatest minds in history had ceased to exist.

Euler's contribution to theoretical aerodynamics were monumental; whereas Bernoulli and d'Alembert made contributions toward physical understanding and the formulation of principles, Euler is responsible for the proper mathematical formulation of these principles, thus opening the door for future quantitative analyses of aerodynamic problems – analyses that continue on to the present day. The governing equations for an inviscid flow, incompressible or compressible, were presented by Euler in a set of three papers: Principles of the Motion of Fluids (1752), General Principles of the State of Equilibrium of Fluids (1753), and General Principles of the Motion of Fluids (1755). The successful derivation of these equations depended on two vital concepts that Euler borrowed in total or in part from previous researchers, as follows:

1. The fluid can be modeled as a continuous collection of infinitesimally small fluid elements moving with the flow, where each fluid element can change its shape and size continuously as it moves with the flow, but at the same time all the fluid elements taken as a whole constitute an overall picture of the flow as a continuum. The modeling of a flow by means of small fluid elements of finite size was suggested by Leonardo; however, the science and mathematics of Leonardo's time were not advanced enough for him to capitalize on this model. Later, Bernoulli suggested that a flow can be modeled as a series of thin slabs perpendicular to the flow; this is not unreasonable for the flow through a duct such as the horizontal pipe at the bottom of Figure 2. However, the thin slab model lacks the degree of mobility that characterizes a small fluid element that can move along a streamline in three dimensions. A major advancement in flow modeling was made by D'Alembert; in 1744 he utilized a moving fluid element to which he applied the principle of mass conservation. Building on these ideas, Euler refined the fluid element model by considering an infinitesimally small fluid element to which he directly applied Newton's second law expressed in a form that utilized differential calculus. Indeed, this leads to the second point.
2. Newton's second law can be applied in the form of the following differential equation, which is a statement that force equals mass times acceleration, that is,

$$F = m\,\mathrm{d}^2x/\mathrm{d}t^2 \tag{11}$$

In this differential equation, F is the force, M is the mass, and $\mathrm{d}^2x/\mathrm{d}t^2$ is the linear acceleration, that is, the second derivative of the linear distance, x. This is today the most familiar form of Newton's second law, it was first formulated in this form by Euler, and was documented in his paper entitled Discovery of a New Principle of Mechanics, published in 1750.

Utilizing the two concepts listed above, namely that of an infinitesimally small fluid element moving along a streamline, and the application of both the principle of mass conservation and Newton's second law to the fluid element in the form of differential calculus as given above, Euler derived the partial differential equations of fluid motion that today carry his name, and that serve as the foundation for a large number of modern aerodynamic analyses. The equations derived by Euler in 1753 revolutionized the analyses of fluid dynamic problems. However, there was one important physical quantity missing from the Euler equations – friction. This leads to our next section.

9 INCLUSION OF FRICTION IN THEORETICAL FLUID DYNAMICS: THE WORKS OF NAVIER AND STOKES

At the beginning of the 19th century, the equations of fluid motion as derived by Euler were well known. However, these equations neglected an important physical phenomenon – a phenomenon that was appreciated by scientists in the 18th and 19th centuries but was not understood well enough to be properly included in any theoretical analysis – namely, friction. The governing flow equations that contain terms to account for friction are called the Navier–Stokes equations, named after the Frenchman Louis Marie Henri Navier (1785–1836) and the Englishman George Gabriel Stokes (1819–1903), who independently derived these equations in the 18th century. More than 150 years later, the Navier–Stokes equations are still the fundamental equations used to analyze a viscous fluid flow. Moreover, they are the subject of much research and application in the field of computational fluid dynamics today. Hence, the importance of the Navier–Stokes equations to modern fluid dynamics cannot be overstated.

The first accurate representation of the effects of friction in the general partial differential equations of fluid flow was given by Navier in 1822, as described in his papers entitled "Memoire sur les lois du mouvement des fluides," presented to the Paris Academie des Sciences. This was published five years later by the Academy. However, although

Navier's equations were of the correct form, his theoretical reasoning was greatly flawed, and it is almost a fluke that he obtained the correct terms. Moreover, he did not fundamentally appreciate the true physical significance of what he had obtained. Before we explore these statements further, let us look at the man himself.

Claude Louis Marie Henri Navier was born in Dijon, France, on 10 February 1785. His early childhood was spent in Paris, where his father was a lawyer to the Legislative Assembly during the French Revolution. After the death of his father in 1793, Navier was left under the care and tutelage of his mother's uncle, the well-known engineer Emiland Gauthey. (At the time of his death in 1806, Gauthey was considered France's leading civil engineer.) As a result of his granduncle's influence, Navier entered the Ecole Polytechnique in 1802, barely meeting the school's admission standards. However, within a year, Navier flowered, and he was among 10 students chosen to work in the field at Boulogne instead of spending his second year at the Polytechnique. In 1804, he entered the Ecole des Ponts et Chaussees, graduating in 1806 near the top of his class. During this time, he was influenced by the famous French mathematician, Jean Baptise Fourier, whom Navier had as a professor of analysis. Fourier's impact on Navier was immediate and lasting. Within a short time, Navier became Fourier's protégé and lifetime friend.

During the next thirteen years, Navier became a scholar of engineering science. He edited the works of his granduncle, who had died in 1806; these works represented the traditional empirical approach to numerous applications in civil engineering. In the process, Navier, based on his own research in theoretical mechanics, added a somewhat analytical flavor to the works of Gauthey. This, in combination with textbooks which Navier wrote independently for practicing engineers, introduced the basic principles of engineering science to a field which heretofore had almost been completely empirical. In fact, Navier is responsible for introducing the precisely defined concept of mechanical work in the analysis of machines. (Navier called the product of force times distance the "quality of action.")

Because of his insistence on the importance and usefulness of engineering science in the solution of practical problems, in 1819 Navier was given a teaching position at the Ecole des Ponts et Chaussees, where he permanently changed the style of teaching in engineering with his emphasis on physics and analyses. In 1831, he replaced the famous mathematician, Augustin Louis de Cauchy at the Ecole Polytechnique. For the rest of his life, Navier lectured at the university, wrote books, and at times practiced his profession of civil engineering particularly in the design of bridges. (It is ironic that the bridge design that brought him the most public notice collapsed before it was totally constructed. This was a suspension bridge over the Seine river in Paris. Toward the end of construction on the bridge, a sewer near one pier ruptured, flooding the area, weakening the foundation of the pier, and causing the bridge to sag. The damage could have been easily repaired. However, for various political and economic reasons, the Municipal Council of Paris had been opposed to building Navier's bridge. The listing of the bridge due to the sewer failure gave the Council the opportunity to lobby for halting the project. The Council was successful, the bridge was torn down, and Navier was greatly disappointed. Here is one of many examples in history where engineering competence is no match for fate and politics – even for a person as well respected as Navier.)

Bridges notwithstanding, history will recognize Navier as the first to derive the governing equations for fluid flow including the effects of friction. However, there is irony here also. Navier had no concept of shear stress in a flow (i.e., the frictional shear stresses acting on the surface of a fluid element). Rather, he was attempting to take Euler's equations of motion and modify them to take into account the forces that act between the molecules in the fluid. He assumed these intermolecular forces to be repulsive at close distance, and attractive at larger distances away from the molecule; thus, for a fluid that is stationary, the spacing between molecules is a result of the equilibrium between the repulsive and attractive forces. Carrying through an elaborate derivation using this model, Navier produced a system of equations that were identical to Euler's equations of motion, except for additional terms that appeared due to the intermolecular forces. For the mathematically versed readers, these terms as derived by Navier involved second derivatives of velocity multiplied by a constant, where the constant simply represented a function of spacing between the molecules. This is indeed the proper form of the terms involving frictional shear stress, namely a second derivative of velocity multiplied by a coefficient called the viscosity coefficient. The irony is that, although Navier had no concept of shear stresses and did not set out to obtain the equations of motion including friction, he nevertheless obtained the proper form of the equations for flow with friction. Later in the 19th century, this form was indeed recognized as proper for frictional flow and that is why the governing equations for flow with friction today are called, in part, the Navier–Stokes equations. However, Navier did not appreciate the true significance of his result; indeed, he did not attribute any physical significance whatsoever to the constant multiplying the second derivatives of velocity – the constant that later was clearly identified as the coefficient of viscosity. (This author notes parenthetically that, in the final analysis, Navier's results were not totally a fluke. Our modern understanding of the physical significance of viscosity coefficient is directly proportional to the molecular mean free path – the mean distance a molecule moves in between successive collisions with other molecules. Hence Navier's approach wherein he was accounting for the spacing between the molecules due to the balance between attractive and repulsive intermolecular forces is not totally off the mark, although the mean free path and the mean spacing between molecules are different values – right church but wrong pew.)

Although Navier did not appreciate the real physical significance of his equations for a fluid flow, one of his contemporaries did: Jean Claude Barre de Saint-Venant. Born in Villiers-en-Biere, Leine de-Marne, France on 23 August 1797, Saint-Venant was educated at the Ecole Polytechnique, graduating in 1816, 12 years after Navier finished at the same school. Saint-Venant then joined the Service des Poudres et Salpetres, and in 1823 moved to the Service des Ponts et Chaussees. Here he served for 20 more years, after which he retired to a life of teaching and research. He died at the age of 92, after a long and productive life, on 6 January 1886, at St. Oven, Loir-et-Cher, France. Saint-Venant was one generation younger than Navier, both in age and professional stature. Navier was elected to the Paris Academy of Sciences in 1824; Saint-Venant became a member in 1868. However, Saint-Venant was quite familiar with Navier's work, as reflected in his book *Mecanique Appliquee de Navier, Annotee par Saint-Venant*, published in Paris in 1858. Seven years after Navier's death, Saint-Venant published a paper at the Academy of Sciences wherein he re-derived Navier's equations for a viscous flow considering internal viscous stresses – eschewing completely Navier's molecular model approach. Appearing in the year 1843, this paper was the first to properly identify the coefficient of viscosity and its role as a multiplying factor with velocity gradients in the flow. He further identified these products as viscous stress acting within the fluid due to the influence of friction. Hence, in 1843, Saint-Venant had gotten it right, and had recorded it. Why it is that his name is never associated with these equations is a mystery to this author, and simply has to be accepted as a miscarriage of technical proprietorship.

This leads up to Sir George Gabriel Stokes, who was just a few hundred miles away from Navier and Saint-Venant, across the English Channel, but who was light years away in terms of familiarity with the work of these Frenchmen. George Stokes is the second-half namesake of the Navier–Stokes equations. Before we examine why, let us first look at the man himself.

Stokes was born in Skreen, Ireland, on 13 August 1819. The hallmark of his family was religious vocations; his father was the rector of the Skreen parish, his mother was the daughter of a rector, and ultimately all of his brothers became ministers of the church. Throughout his life, George Stokes remained a strongly religious person. Indeed, toward the end of his life, he became interested in the relationship of science to religion; from 1886 to the year of his death in 1903, he was president of the Victoria Institute of London, a society for examining the relationship between Christianity and contemporary thought, with emphasis on science. During his childhood, Stoke's education began with tutoring from his father, which led to his admission to Bristol College in Bristol, England. At Bristol, he prepared for university studies, and entered Pembroke College, Cambridge, at the age of 18. Stokes was a highly intelligent man; at the time of graduation from Cambridge, he was immediately elected to a fellowship in Pembroke College. Eight years later, Stokes occupied the Lucasian Chair at Cambridge, the same professorship held by Newton almost two centuries earlier. Since the Lucasian endowment was small, Stokes had to simultaneously take a second position in the 1850s, teaching at the Government School of Mines in London. He held the Lucasian Chair until death at Cambridge on 1 February 1903.

Fluid dynamicists think of George Stokes and they visualize a man who made a momentous, fundamental contribution to the discipline via his derivation and subsequent use of the equations, which today are called the Navier–Stokes equations. These equations are the most fundamental descriptors of a general three-dimensional, unsteady, viscous fluid flow; they are the foundation of modern theoretical and computational fluid dynamics. However, if Stokes was alive today, he would most likely feel more comfortable in being identified as a physicist and to some small degree a mathematician who had made substantial contributions in the area of optics. Beginning about 1845, he worked on the propagation of light and how it interacted with the ether – a continuous substance surrounding the earth according to the prevailing theory of that day. It is interesting to note that Stokes analyzed the properties of the hypothetical ether using an analogy with his fluid dynamic equations of motion. He concluded that if the earth moved through a stationary ether, the ether must be a very rarefied fluid. In a contradictory sense, he also concluded that the propagation of light required the ether to be much like a very elastic solid. Hence, one of the first theoretical consequences of the Navier–Stokes equations was not a definitive flowfield calculation (as used today), but rather an inconclusive study of the properties of the ether. To make things more inconclusive, Stokes showed in 1846 that the laws of reflection and refraction remained unchanged whether or not an ether existed. Of much greater importance in the physics of light was Stokes' work on fluorescence, the phenomenon wherein a substance absorbs electromagnetic waves on one wavelength, and emits waves of another wavelength. In particular, he made observations of the blue light emitted from the surface of an otherwise transparent and colorless solution of sulfate of quinine when the solution is irradiated by invisible ultraviolet rays. His physical explanation of this process won him the Rumford Medal of the Royal Society in 1852; indeed, he coined the word "fluorescence" in the context of his explanation. Later, he suggested the use of fluorescence to study the properties of molecules and is credited as the first to develop the principles of spectrum analysis. In summary, the point made here is that Sir George Stokes would most likely credit himself for contributions in optics rather than fluid dynamics. In this sense, there is some irony in the fact that today his name is literally invoked by fluid dynamicists much more frequently than by those working in any other field of science and engineering.

With this as background, we now focus on Stoke's contributions in fluid dynamics. He was unfamiliar with the work of Navier and Saint-Venant in France, and was not aware of their derivations of the equations of motion for a fluid with friction. Quite independently, he utilized the concept of internal shear stresses in a moving fluid, and derived the governing equations of a viscous fluid (a fluid with internal friction). His derivation of the equations was much like the way they are derived today; in the process, he properly identified the dynamic viscosity coefficient, μ, as it appears in the Navier–Stokes equations. This work was published in 1845 (two years after Saint-Venant's similar derivation) in his paper entitled "On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids." As with most scientists studying fluid dynamics in the 19th century, Stokes dealt with an incompressible flow. For such flows, the energy equation is not essential. With this one exception, the work of Stokes remains unchanged to the present day. The fundamental equations for a flow with friction – the Navier–Stokes equations – were therefore well established more than 150 yr ago. This should be a sobering thought for modern fluid dynamicists, and especially for those at the cutting edge of modern computational fluid dynamics, who deal with the Navier–Stokes equations on an almost daily basis. Here, we are using ultramodern supercomputers to solve equations that are covered by the dust of ages, but that have nonetheless weathered the test of time.

10 OSBORNE REYNOLDS: UNDERSTANDING TURBULENT FLOW

There are two types of viscous flows: *laminar flow*, in which the fluid elements move in a regular ordered fashion and adjacent streamlines move smoothly over each other as if they were part of a medium made up of different well-ordered laminae, and *turbulent flow*, in which the fluid elements move in a disordered fashion and the streamlines form a tortuous mixed-up, irregular pattern. The viscous stresses that cause skin-friction drag on a body are higher for turbulent flow than for laminar flow. Hence, it is vital to know whether the flow is laminar or turbulent. In reality, a viscous flow generally starts out as laminar and then undergoes a transition to turbulent flow. Unfortunately, an understanding of the fundamental nature of turbulent flows sufficient to allow accurate predictions of their properties is still today one of the unsolved problems of classical physics. However, an important first step in the study of the transition from laminar to turbulent flow was taken in the latter part of the nineteenth century by Osborne Reynolds. His pioneering studies were the foundation of over 150 yr of constant and intensive research on turbulent flows – research that still continues unabated today.

Osborne Reynolds (1842–1912) was born on October 23, 1842 in Belfast, Ireland. He was raised in an intellectual family atmosphere; his father had been a fellow of Queens' College. Cambridge, a principal of Belfast Collegiate School, headmaster of Dedham Grammar School in Essex, and finally rector at Debach in Suffolk. Already in his teens Reynolds showed intense interest in the study of mechanics and appeared to have a natural aptitude. At the age of 19, he served a short apprenticeship in mechanical engineering before entering Cambridge University a year later. Reynolds was a highly successful student, graduating with the highest honors in mathematics. In 1867 he was elected a fellow of Queens' College.

In 1868, Owens College (later the University of Manchester) established its chair of engineering, the second such chair in an English university (the first had been the chair of civil engineering at the University College, London, in 1865). Reynolds applied for the Owens chair, writing in his application that "from my earliest recollection I have had an irresistible liking for mechanics and the physical laws on which mechanics as a science is based. In my boyhood I had the advantage of the constant guidance of my father, also a lover of mechanics and a man of no mean attainment in mathematics and their application to physics." Despite his youth and relative lack of experience, Reynolds was appointed to the chair at Manchester, where he remained until his retirement in 1905.

During his 37 years at Manchester, Reynolds distinguished himself as one of the leading practitioners of classical mechanics. He worked on problems involving electricity, magnetism, and the electromagnetic properties of solar and cometary phenomena. After 1873, he focused on fluid mechanics – the area in which he would make his most importance contributions.

Reynolds was a scholarly man, with high standards. Engineering education was new to English universities at that time, and Reynolds had definite ideas about its proper form. He believed that all engineering students, no matter what their specialty, should have a common background based in mathematics, physics, and particularly the fundamentals of classical mechanics. He organized a systematic engineering curriculum at Manchester covering the basics of civil and mechanical engineering. Despite his intense interest in education, he was not a great lecturer. His lectures were difficult to follow, and he frequently wandered among topics with little or no connection. He was known to stumble upon new ideas during the course of a lecture and to spend the remainder of the time working out those ideas at the blackboard, oblivious to his students. He did not spoon-feed his students, and many did not pass his course, but the best students enjoyed his lectures and found them stimulating, such as J.J. Thomson, who in 1906 received the Nobel Prize in physics for demonstrating the existence of the electron.

In regard to Reynolds' emphasis on a research approach, his student and colleague, Professor A. H. Gibson, commented as follows in his biography of Reynolds, written for the British Council in 1946: "Reynolds's approach to a problem was essentially individualistic. He never began by reading what others thought about the matter, but first thought this out for himself. The novelty of his approach to some problems made some of his papers difficult to follow, (but his) more descriptive physical papers . . .. make fascinating reading, and when addressing a popular audience, his talks were models of clear exposition."

At the turn of the century, Reynolds' health began to fail, considerably diminishing his physical and mental capabilities; a particularly sad state for such a brilliant scholar. He died at Somerset, England, in 1912. Sir Horace Lamb, noted researcher in fluid dynamics and a longtime colleague of Reynolds, commented as follows:

"The character of Reynolds was like his writings, strongly individual. He was conscious of the value of his work, but was content to leave it to the mature judgment of the scientific world. For advertisement he had no taste, and undue pretension on the part of others only elicited a tolerant smile. To his pupils he was most generous in the opportunities for valuable work which he put in their way, and in the share of cooperation. Somewhat reserved in serious or personal matters and occasionally combative and tenacious in debate, he was in the ordinary relations of life the most kindly and genial of companions" (obituary, by Horace Lamb, Proceedings of the Royal Society, ser. A, vol. 88, 24 February 1913).

Reynolds' three contributions to fluid mechanics were pivotal and seminal. The first was his study of the transition from laminar flow to turbulent flow in pipes. To put that contribution in perspective, we must fall back two decades and examine the work of a German hydraulics engineer, Gotthilf Heinrich Hagen (1797–1884). Hagen was the first to report that two distinct types of flow could exist inside pipes, hinting of that situation in a closing remark in a paper published in 1839. Concerning the flow of water in pipes, he referred to "strong movement" that the water demonstrated under certain flow conditions. He went on to express a certain degree of frustration: "The exact investigation of the results produced in this case appears hence to offer great difficulties; at least I have not yet succeeded in clarifying sufficiently the peculiarities which are then evidenced." The strong movements observed by Hagan were associated with what today we call a turbulent flow. A more graphic description was given by Hagen in a paper published in 1855, discussing the effects of heating a tube through which water was flowing. The tubes were made of glass to enable him to observe the nature of the flow:

"Since I invariably had the efflux jet before my eyes, I noticed that its appearance was not always the same. At small temperatures it remained immovable, as though it was a solid glass rod. On the other hand, as soon as the water was more strongly heated, very noticeable fluctuations of short period were established, which with further heating were reduced but nevertheless even at the highest temperatures did not wholly disappear . . . With each repetition of the experiment the same phenomenon occurred, and when I finally made the graphic summary, I found that the strongest fluctuations always took place in that portion of the curve where the velocity decreased with increasing temperature. . .."

"Special (observations) that I made with glass tubes showed both types of movement very clearly. When I let sawdust be carried through with the water, I noticed that at low pressure it moved only in the axial direction, whereas at high pressure it was accelerated from one side to the other and often came into whirling motion."

From the perspective of modern aerodynamics, we understand what happened in Hagen's experiment: Laminar flow was destabilized by addition of heat to the flow. In Hagen's experiment, at low temperatures the water flow through the small glass tube was a laminar flow which was stable. Because heat was added, thus increasing the flow temperature, the laminar flow was shifted from a stable regime to an unstable regime. Given even a slight disturbance, that heated, unstable laminar flow easily made the transition to turbulent flow, just as Hagen described.

Hagen did not determine quantitative criteria for the conditions at which the transition from laminar flow to turbulent flow would occur. That was where Reynolds' contribution became so important. In 1883, Reynolds reported his findings from a series of fundamental experiments that would have lasting effects for analyses of where the transition from laminar flow to turbulent flow would occur. His work, like that of Hagen, showed that there could be two distinct types of viscous flow – laminar and turbulent – but Reynolds' experiments were better controlled and better designed for quantification than those of Hagen. Reynolds' experimental apparatus is shown in Figure 4, from Reynolds' original paper. Reynolds filled a large reservoir with water that fed into a glass pipe through a larger bell-mouth entrance. As the water flowed through the pipe, he introduced dye into the middle of the stream at the entrance of the bell mouth. Figure 5 (also from Reynolds' original paper) shows what happened to that thin filament of dye as it flowed through the pipe. The flow was from right to left. If the flow velocity was low, the thin dye filament would travel downstream in a smooth, neat, orderly fashion, with clear demarcation between the dye and the rest of the water (Figure 5a). If the flow velocity was increased beyond a certain value, the dye filament would suddenly become unstable and fill the entire pipe with color (Figure 5b). Reynolds clearly pointed out that the smooth dye filament corresponded to laminar flow in the pipe, whereas the agitated and totally diffused dye filament was due to turbulent flow in the pipe. Furthermore, he studied the details of that turbulent flow by visually observing the pipe flow illuminated by a momentary electric spark, much as we would use a strobe light today. He saw that the turbulent flow consisted of a large number of distinct eddies (Figure 5c). The transition from laminar flow to turbulent flow occurred when the parameter defined by $\rho VD/\mu$ exceeded a certain critical value, where ρ was the density of the water, V was the mean flow velocity, μ was the viscosity coefficient, and D was the diameter of the pipe. That dimensionless parameter, first introduced by Reynolds, would become known as the Reynolds number. Reynolds determined that the critical value for that parameter, the value above which turbulent flow would occur, was 2300. It was indeed a fundamental finding: It indicated that the transition phenomenon did not depend simply on velocity by itself, nor on density by itself, nor on the size of the flow by itself, but rather on the particular combination of the variables defined earlier that make up the Reynolds number. No matter what the velocity or density or viscosity of the flow, and no matter what the size of the pipe through which the flow was moving, transition would occur, according to Reynolds' calculations, at a value of 2300 for the combination $\rho VD/\mu$. That was a stunning discovery. Accurate determination of where on a surface the transition from laminar flow to turbulent flow will occur is perhaps the highest priority in modern aerodynamics, and the use of Reynolds numbers for that determination is still the approach used today.

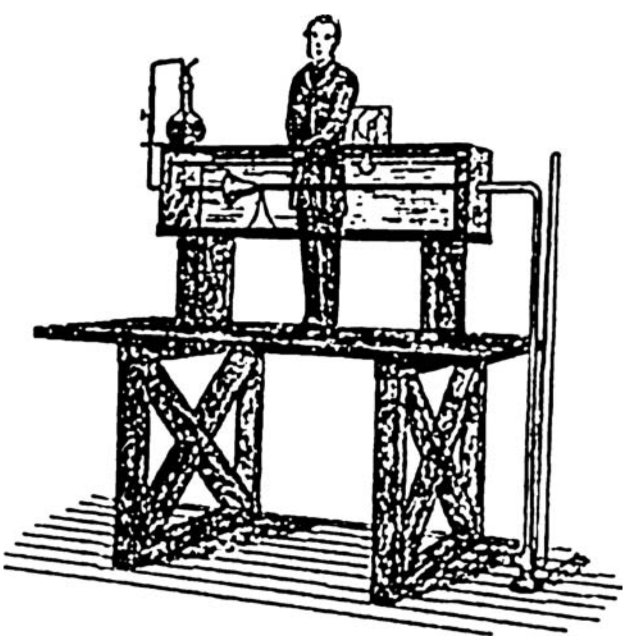

Figure 4. Reynolds experimental apparatus for studying transition (1883).

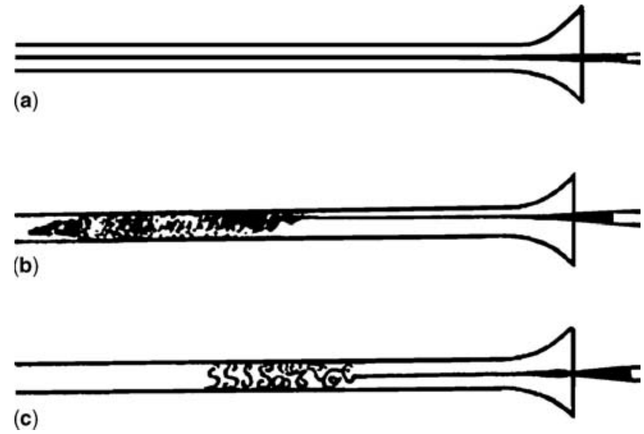

Figure 5. Reynolds sketches of the transition phenomena for flow in a pipe.

Reynolds' second major contribution was the conception and implementation of a theoretical model for analysis of a turbulent flow – for detailed calculation of the velocity, density, and temperature distributions throughout a turbulent flow field. The governing equations for the flow-field variables in a viscous flow are the Navier–Stokes equations. Solution of these equations will give, in principle, the variations of the flow-field properties over the whole x–y–z space as functions of time. For simplicity, let us consider a steady flow, where the flow-field variables at all points in the flow are independent of time. This presupposes a steady, laminar flow, for such a flow there are no fluctuations at any given point. Reynolds sketched turbulent, fluctuating flow in Figure 5c. No matter how small the eddies, a turbulent flow locally at any given point is an unsteady flow. In a turbulent flow, if we lock our attention onto a given point, we will see that the local flow-field values are changing as a function of time. However, Reynolds theorized that if one took a suitable time average for each flow property in a turbulent flow, that time average would be a steady value. Taking a hint from some methods in the kinetic theory of gases, Reynolds specifically assumed that each variable in a turbulent flow was locally composed of its time mean, say $\bar{u}$, and its time wise fluctuating component, u', such that the actual local value at any instant in time would be expressed as $u = \bar{u} + u'$. Moreover, the Navier–Stokes equations can be assumed to hold if the dependent variables (p, ρ, u, v, T, etc.) that appear in those equations are interpreted as their time-averaged values. However, when the time averaging of those equations is done mathematically, some extra terms appear in the equations that can be interpreted as a "turbulent viscosity" μ_T and a turbulent thermal conductivity k_T. Therefore, when the Navier–Stokes equations are to be used to study a turbulent flow, according to Reynolds the flow properties are to be used as their time averages, and the viscosity coefficient and the thermal conductivity are to be replaced by the sums ($\mu + \mu_T$) and ($k + k_T$), respectively, where μ_T and k_T are the apparent increases in viscosity and thermal conductivity due to the fluctuating, turbulent eddies, respectively. With this formalism, the Navier–Stokes equations become the Reynolds-averaged Navier–Stokes equations for turbulent flow – a set of equations used in what is today by far the most frequently employed theoretical approach to engineering analyses of turbulent flows. That method of treating a turbulent flow locally as the sum of a time-averaged mean and a fluctuating component was the most substantial and pivotal of Reynolds' contributions to fluid dynamics, and its impact on aerodynamics has been historic. The vast majority of theoretical predictions of skin-friction drag on aerodynamic shapes have used, in one form or another, the time-averaged model of Reynolds.

Reynolds' theoretical model, important as it was, did not "solve" the problem of turbulence. The Reynolds-averaged Navier–Stokes equations introduced the turbulent viscosity μ_T and turbulent thermal conductivity k_T. In any analysis of turbulent flow, we need appropriate numbers for μ_T and k_T, and that can be a big problem, for such values will depend on the nature of the flow itself. In direct contrast, the values for μ and k (the molecular viscosity and molecular thermal conductivity) are known properties of the fluid that can be looked up in standard reference sources. Finding the proper values for μ_T and k_T for a given turbulent flow is called turbulence modeling. Reynolds introduced his time-averaged equations in 1894, but today, 115 years later, research to find the best, most appropriate turbulence models to calculate for μ_T and k_T is one of the highest priorities in aerodynamics.

Reynolds' third contribution of significance for aerodynamics, though of lesser importance than the two discussed earlier, was in determining the connection between skin friction and heat transfer. Today, there is an approximate relation used by engineers that relates the local skin-friction coefficient C_F to the local heat-transfer coefficient C_H via the Reynolds analogy, which can be written as

$$\frac{C_H}{C_F} = f(Pr) \tag{12}$$

where $f(Pr)$ denotes a function of the Prandtl number ($Pr = \mu c_p/k$), with c_p being the specific heat at constant pressure). First introduced by Reynolds in 1874, the Reynolds analogy has come into its own since the middle of the twentieth century, when aeronautical engineers had to begin to cope with the problems of aerodynamic heating associated with supersonic and hypersonic flight.

Many a contribution in the physical sciences has had a certain half-life, with diminishing importance as the years have gone by, but Reynolds' contributions, viewed in the light of modern aerodynamic applications, have actually increased in significance. The entire field of modern turbulence modeling and even our basic views of the nature of turbulence and transition have derived from the ideas of Reynolds.

11 THE CIRCULATION THEORY OF LIFT: KUTTA AND JOUKOWSKI

In 1902 the Wright Brothers were conducting wind-tunnel tests in Dayton, Ohio, which advanced applied aerodynamics toward maturity and helped in the design of their successful 1903 Wright Flyer. At the same time, Wilhelm Kutta was finishing some work at the University of Munich that would prove an important advance in theoretical aerodynamics. Kutta was born in Pitschen, Germany, in 1867. In 1902, at the age of 35, he received a Ph.D. in mathematics from Munich, with a dissertation on aerodynamic lift. Kutta's interest had been sparked by the glider flights of Otto Lilienthal between 1890 and 1896 in Germany. Kutta knew that Lilienthal had used a cambered airfoil for his gliders. Moreover, he knew that when the cambered airfoil was put at a zero angle of attack, positive lift was still produced. That was clearly evident from Lilienthal's data. Indeed, all of the cambered airfoils tested by Lilienthal had to be pitched to some negative angle of attack in order to reach the point of no lift (the zero-lift angle of attack). The generation of lift by a cambered airfoil at a zero angle of attack was counterintuitive to many mathematicians and scientists at that time, but the experimental data unequivocally indicated it to be a fact. Such a mystery made the theoretical calculation of lift on a cambered airfoil an excellent research topic at the time, and Kutta eagerly took it on. By the time he finished his dissertation in 1902, Kutta had in hand the first mathematical calculations of lift on cambered airfoils. Although not seen explicitly in his calculations, his results linked lift to circulation (see Governing Equations for Fundamental Aerodynamics). However, in a paper given to the Royal Bavarian Academy of Sciences in January, 1910, by reinterpreting some of his 1902 theoretical development, he found the classic relation for lift as the product of density, velocity, and circulation, albeit in a form slightly different from that buried in the 1902 dissertation. For that reason, it can be said that Kutta shared in the development of the circulation theory of lift. However, that came to light only in 1910, five years after the relation was clearly and explicitly derived and published independently by Nikolai Joukowski in Moscow.

Kutta was primarily a mathematician whose interest in aerodynamics was sparked by Lilienthal's glider flights. After 1902 he was a professor of mathematics, finally settling at the Technische Hochschule in Stuttgart in 1911, from where he retired in 1935. His death came in 1944 as Germany was rushing headlong into defeat in World War II.

At the time the Wright brothers were carrying out their wind-tunnel tests and Kutta was finishing his dissertation, a 55-year-old professor was directing the construction of the first wind tunnel in Russia. Nikolai Joukowski (Zhukovsky) was professor of mechanics at Moscow University and professor of mathematics at the Moscow Higher Technical School, at which the wind tunnel was being built. A native of Orekhovo, Vladimirprovine, Russia, Joukowski was born on 17 January 1847, the son of a communications engineer. Joukowski earned a bachelor's degree in mathematics from the University of Moscow in 1868 and in 1870 began teaching. In 1882 he completed his Ph.D. at Moscow University with a dissertation on the stability of fluid flows. Four years later he became the Head of the Department of Mechanics of Moscow University. Joukowski published more than 200 papers in his lifetime, dealing with basic and applied mechanics. By the turn of the century he was one of Russia's most respected scientists; considered the founder of Russian hydrodynamics and aerodynamics. In 1885 he was awarded the N.D. Brashman Prize for major theoretical research in fluid dynamics, and in 1894 he became a member of the St. Petersburg Academy of Sciences. From 1905 until his death in 1921, Joukowski served as president of the Moscow Mathematical Society.

In the late 1880s, contemporary with Otto Lilienthal's aeronautical activities, Joukowski became interested in flying machines. In 1895 he visited Lilienthal in Berlin and purchased one of the eight gliders that Lilienthal sold to the public. This was the first time that a university-educated mathematician and scientist had become closely connected with a real flying machine, actually getting his hands on one. Joukowski was motivated by his interest in flying machines to examine the aerodynamics of flight on a theoretical, mathematical basis.

In particular, he directed his efforts toward the calculation of lift. As early as 1890 he began to conceive a model of the flow over a lifting airfoil as consisting in some way of vertical motions caused by the fluid viscosity. He envisioned bound vortices fixed to the surface of the airfoil, along with the resulting circulation that somehow had to be related to the lifting action of the airfoil. Finally, in 1906 he published two notes, one in Russian and the other in French, in two rather obscure journals: *Transactions of the Physical Section of the Imperial Society of Natural Sciences*, in Moscow, and *Bulletin de l'Institut Aerodynamique de Koutchino*, in St. Petersburg. In those notes he derived and used the following relation for calculating the lift per unit span of an airfoil:

$$L = \rho V \Gamma \tag{13}$$

where Γ is the circulation, a technically defined quantity equal to the line integral of the flow velocity taken around any closed curve encompassing the airfoil. This equation was a revolutionary development in theoretical aerodynamics, for the first time allowing calculation of the lift on an airfoil with mathematical precision. Because Kutta was able to show in hindsight that the essence of that relation could be found buried in his 1902 dissertation, this equation has become known as the Kutta–Joukowski theorem. It is still taught in university aerodynamics courses and is used to calculate the lift for airfoils in low-speed incompressible flows.

The apparent simplicity of the Kutta–Joukowski theorem belies the fact that considerable effort usually is required to calculate the value of Γ for a given airfoil at a given angle of attack in a free stream of a given velocity. This is where the model of vortex filaments aligned with the span of the wing comes into the picture. The strengths of the vortex filaments must be calculated precisely so that the resulting flow (the flow induced by the vortices plus the flow due to the free stream) will be tangent to the airfoil along its surface. Once the proper vortex strengths are calculated, they are added together to yield the total circulation Γ associated with the complete airfoil. That is the value of Γ that is inserted into the Kutta–Joukowski theorem to give the lift per unit span.

The circulation theory of lift provided the foundation for all theoretical aerodynamics for the first 40 yr of the twentieth century, after which the advent of high-speed flight required that the compressibility of air be taken into account. The circulation theory of lift is still alive and well today, for example, it is the basis for modern "panel" techniques, carried out with digital computers, for calculating lift values for airfoils in inviscid, incompressible flows. Such panel techniques are continually being revised and improved, and thus the circulation theory of lift is still evolving today, more than 100 yr after its first introduction.

Joukowski went on to become "the father of Russian aviation." He established an aerodynamics laboratory in Moscow during the first decade of the twentieth century and gave a series of lectures on the theoretical basis of aerodynamics, relying heavily on his own theoretical and experimental work – the first systematic course in theoretical aerodynamics. Those lecture notes were recorded by two of his students and were published after Joukowski reviewed them. The first Russian edition appeared in 1912, and the first French edition in 1916; second editions in Russian and in French confirmed the value of those notes. Joukowski developed a means of designing airfoils using conformal mapping and the techniques of complex variables. Those Joukowski airfoils were actually used on some aircraft, and today those techniques provide a mathematically rigorous reference solution to which modern approaches to airfoil design can be compared for validation. During World War I, Joukowski's laboratory was used as an instructional school for new military pilots. Shortly before his death, Joukowski founded a new aerodynamics laboratory just outside Moscow called the Central Institute for Aerodynamics. This institute continues to the present time, known as TsAGI, it is Russia's premier aerodynamics facility, the Russian equivalent of the NASA laboratories.

Nikolai Joukowski died in Moscow, 17 March 1921. At the time of his death, he was working in the two areas of high-speed aerodynamics and aircraft stability. Always moving forward, the man who revolutionized the analysis of low-speed airfoils in 1906 would exit the world in 1921 with his attention fixed on analysis of supersonic vehicles and the wave patterns associated with such objects.

12 LUDWIG PRANDTL AND HIS BOUNDARY-LAYER THEORY

Until 1904, the role that friction played in determining the characteristics of the flow over a body was conjectural and somewhat controversial. There was the question of what happened at the surface of the body: Did the fluid immediately adjacent to the surface stick to the surface, giving zero velocity at the surface, or did it slip over the surface with some finite velocity? And there was always the question as to how much of the flow field itself was dominated by the effect of friction.

In 1904, Ludwig Prandtl (1875–1953) read a paper before the Third International Mathematical Congress at Heidelberg that was to bring revolutionary changes in aerodynamics (Prandtl, 1904). It was only eight pages long, but it would prove to be one of the most important fluid-dynamics papers ever written. Much later, in 1928, when asked by the fluid dynamicist Sydney Goldstein why the paper was so short, Prandtl replied that he had been given only 10 min for his presentation, and he was under the impression that his paper could contain only what he had time to say.

The important thing about Prandtl's paper was that it gave the first description of the boundary-layer concept. Prandtl theorized that the effect of friction was to cause the fluid immediately adjacent to the surface to stick to the surface (i.e., he assumed the no-slip condition at the surface) and that the effect of that friction was experienced only in the near vicinity of the surface (i.e., the influence of friction was limited to a thin region called the boundary layer). Outside the boundary layer, the flow was essentially uninfluenced by friction (i.e., it was the inviscid potential flow that had been studied for the past two centuries). The concept of the boundary layer is sketched in Figure 6. In the types of flows associated with a body in flight, the boundary layer is very thin compared with the size of the body, much thinner than can be shown in Figure 6a. In Figure 6b, a portion of the boundary layer is enlarged to illustrate the variation of the flow velocity through the boundary layer, going from zero at the surface to the full inviscid-flow value at the outer edge of the boundary layer. With Figure 6 in mind, consider Prandtl's description of the boundary layer:

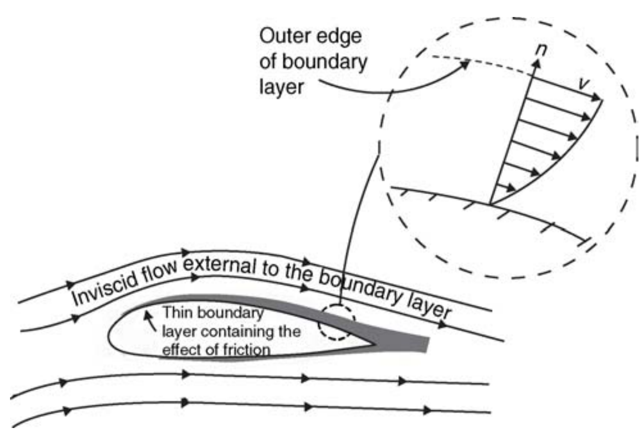

Figure 6. Division of a flow into two parts: (a) the thin boundary layer adjacent to the surface, where the effects of friction are dominant, and an inviscid eternal flow outside of the boundary layer; (b) enlarged sketch of the boundary layer showing the variation in velocity across the boundary layer as a function of the normal distance, perpendicular to the surface.

"A very satisfactory explanation of the physical process in the boundary layer (Grenzschicht) between a fluid and a solid body could be obtained by the hypothesis of an adhesion of the fluid to the walls, that is, by the hypothesis of a zero relative velocity between fluid and wall. If the viscosity was very small and the fluid path along the wall not too long, the fluid velocity ought to resume its normal value at a very short distance from the wall. In the thin transition layer (Uebergangschicht) however, the sharp changes of velocity, even with small coefficient of friction, produce marked results."

One of these "marked results" is that within the boundary layer, there is an enormous change in velocity over a very short distance, as sketched in Figure 6b (i.e., there are very large velocity gradients in the boundary layer). In turn, as described by Newton's shear-stress law, which states that the shear stress is proportional to the velocity gradient, the local shear stress can be very large within the boundary layer.

Another "marked result" is flow separation:

"In given cases in certain points fully determined by external conditions, the fluid flow ought to separate from the wall. That is, there ought to be a layer of fluid which, having been set in rotation by the friction on the wall, insinuates itself into the free fluid, transforming completely the motion of the latter, and therefore playing there the same part as the Helmholtz surfaces of discontinuity."

Prandtl was referring to the type of flow sketched in Figure 7, where the boundary layer separates from the surface. As seen in Figure 7, driven by inviscid-flow conditions of a certain type, the boundary layer can separate and then trail downstream, much like the nineteenth-century concept of a surface of discontinuity. An essentially dead-air region is formed in the wake behind the body. The pressure distribution over the surface of the body is radically changed when the flow separates, such that the altered pressure distribution creates a large unbalanced force in the drag direction – the pressure drag due to flow separation. When there is a massive flow separation (Figure 7), the pressure drag usually is much larger than the skin-friction drag. The type of external inviscid flow that promotes boundary-layer separation is a flow that produces an adverse pressure gradient (i.e., an increasing pressure in the flow direction). Prandtl explained that effect as follows:

"On an increase of pressure, while the free fluid transforms part of its kinetic energy into potential energy, the transition layers instead, having lost a part of their kinetic energy (due to friction), have no longer a sufficient quantity to enable them to enter a field of higher pressure, and therefore turn aside from it."

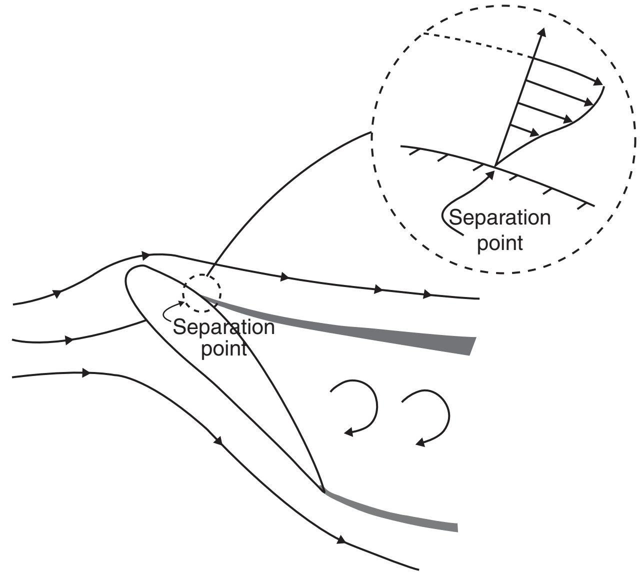

Figure 7. Schematic of separated flow over the top surface of an airfoil at a very high angle of attack – beyond stall.

That phenomenon is illustrated in Figure 7. At the separation point, the fluid elements deep inside the boundary layer (which have already had substantial portions of their initial kinetic energies dissipated by friction) cannot work their way uphill against a region where the pressure is increasing. Hence, the velocity profile is depleted near the surface. At the separation point, it has an inflection point at the surface, as sketched in Figure 7. Beyond that point, the fluid elements near the surface would actually be pushed backward by the increasing pressure, but nature does not allow that to happen; instead, the boundary layer simply lifts off the surface at that point, as shown in Figure 7.

The overall perspective set forth by Prandtl in his 1904 paper was simple and straightforward, namely, that an aerodynamic flow over a body can be divided into two regions: a thin boundary layer near the surface, where friction is dominant, and an inviscid flow external to the boundary layer, where friction is negligible. There is a strong effect of the outer inviscid flow on the boundary- layer properties; indeed, the outer flow is what drives the boundary layer. On the other hand, the boundary layer is so thin that it has virtually no effect on the outer inviscid flow. The exception to that is when the flow separates; then the outer inviscid flow is greatly modified by the presence of the separation region. Prandtl's view of those phenomena was as follows:

"While dealing with a flow, the latter divides into two parts interacting on each other; on one side we have the "free fluid," which (is) dealt with as if it were frictionless, according to the Helmholz vortex theorems, and on the other side the transition layers near the solid walls. The motion of these layers is regulated by the free fluid, but they for their part give to the free motion its characteristic feature by the emission of vortex sheets."

Prandtl used the terms "transition layer" and "boundary layer" interchangeably. Indeed, he used the term "boundary layer" only once in that paper, while frequently referring to the "transition layer." "Boundary layer" is the term that has survived.

Prandtl's 1904 paper is arguably the most important and influential paper published in fluid mechanics in the last century. It may continue to be such in present century. His boundary-layer concept revolutionized fluid dynamics in general and aerodynamics in particular.

Ludwig Prandtl was born on 4 February 1874, in Freising, Bavaria. His father, Alexander Prandtl, was a professor of surveying and engineering at the agricultural college at Weihenstephan, near Freising. Although the Prandtls had three children, two died at birth, and Ludwig grew up as an only child. His mother suffered from a protracted illness, and partly as a result of that he became very close to his father. At an early age he became interested in his father's books on physics, machinery, and instruments. Perhaps his remarkable ability to go straight to the heart of a physical problem can be traced to his childhood environment, for his father, a great lover of nature, taught him to observe phenomena and to reflect on them.

In 1894 Prandtl began scientific studies at the Technische Hochschule in Munich, where his principal teacher was the well-known mechanics professor August Foppl. Six years later he graduated from the University of Munich with a Ph.D., with Foppl as his advisor. By that time Prandtl was alone; his father had died in 1896, and his mother in 1898.

Prandtl showed no interest in fluid mechanics prior to 1900. Indeed, his Ph.D. work at Munich had been in solid mechanics – unstable elastic equilibrium in which bending and distortion acted together. Prandtl continued his interest and research in solid mechanics through most of his life but that work was overshadowed by his many major contributions to the study of fluid flows. Soon after graduation from Munich, Prandtl had his first major encounter with fluid mechanics. Joining the Nurnberg works of the Maschinenfabrik Augsburg as an engineer, Prandtl worked in an office designing mechanical equipment for the new factory. He was assigned to redesign a suction device to collect lathe shavings. Finding no reliable information in the scientific literature on the fluid mechanics of suction, Prandtl carried out some experiments to answer a few fundamental questions about such flows. The result of that work was his new design for a shavings collector. The apparatus was modified with pipes of improved shapes and sizes, and it operated well at one-third of its original power consumption. Prandtl's contributions in fluid mechanics had begun.

A year later, in 1901, Prandtl became a professor of mechanics in the Mathematical Engineering Department at the Technische Hochschule in Hannover (a German "technical high school" is equivalent to a technical university in the United States). At Hannover he developed his boundary-layer theory and began work on supersonic flows through nozzles. After delivering his famous paper on the concept of the boundary layer in 1904, Prandtl's star would rise meteorically. Later that year he moved to the prestigious University of Gottingen to become Director of the Institute for Technical Physics, spending the remainder of his life there and building his laboratory into the greatest aerodynamics research center of the 1904–1930 period.

In 1925, the Kaiser-Wilhelm-Institut fur Stromungsforschung (Kaiser Wilheim Institute for Flow Investigation) was built on the grounds of the Gottingen University, with Prandtl as director, in recognition of his important research achievements in mechanics. By the 1930s, Prandtl was recognized worldwide as the elder statesman of fluid dynamics. He continued to do research in various areas, including structural mechanics and meteorology, but his great contributions to fluid dynamics had already been made. He remained at Gottingen throughout World War II, engrossed in his work and seemingly insulated from the politics of Nazi Germany and the privations and destruction of the war. In fact, the German Air Ministry provided new equipment and financial support for Prandtl's laboratory. Prandtl's attitude at the end of the war, however, was reflected in his comments to a U.S. Army interrogation team at Gottingen in 1945: He complained about bomb damage to the roof of his house, and he asked to what extent the Americans planned to support his current and future research. Prandtl was 70 at the time, and still going strong. However, Prandtl's laboratory did not fare well after the war. Some of his research equipment was dismantled by the Allies, and most of his research staff left Germany, some eventually going to work in the United States and England.

Prandtl died in 1953. He was clearly the father of modern aerodynamics and a monumental figure in fluid dynamics. The impact of his work will reverberate for centuries to come.

13 SUMMARY

With our discussion of Ludwig Prandtl, we end this chapter on the history of the early evolution of theoretical and experimental fluid dynamics. We have just scratched the surface. Moreover, we have not touched the exponential growth of fluid dynamics and aerodynamics that has taken place in the past 100 years. Nevertheless, I hope that you have gained some appreciation for the intellectual accomplishments and the major players that have helped to set the stage for modern aerospace engineering as we know it today, and as you will enjoy reading about throughout this encyclopedia. And if this chapter wets your appetite to find out more about the history of aerodynamics and aerospace engineering (see Anderson, 1997; Anderson, 2002).

REFERENCES

Anderson, J.D. Jr. (1997) *A History of Aerodynamics and its Impact on Flying Machines*, Cambridge University Press, New York.

Anderson, J.D. Jr. (2002) *The Airplane: A History of Technology*, American Institute of Aeronautics and Astronautics, Reston, VA.

Anderson, J.D. Jr. (2008) *Introduction to Flight*, 6th edn, McGraw-Hill, New York.

Prandtl, L. (1904) Ueber Flussigkeitsbeweging bei sehr kleiner Reibung. *Proceedings of the 3rd International Mathematical Congress, Heidelberg, 489–491, Leipzig*.

Rouse, H. and Ince, S. (1957) *History of Hydraulics*, Iowa Institute of Hydraulic Research, University of Iowa, Iowa City, Iowa.

Tokaty, G.A. (1971) *A History and Philosophy of Fluid Mechanics*, G.T. Foulis & Co, Ltd, Henly-on-Thames, Oxfordshire, England.